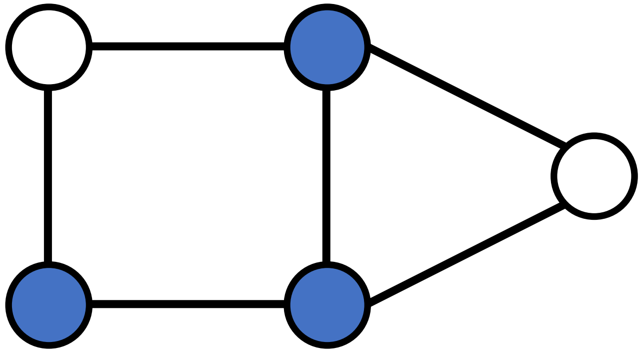
Vertex Cover
CSCI 532

Vertex Cover

Vertex Cover: Given graph, find the smallest subset of vertices such that every edge in the graph has at least one vertex in the subset.



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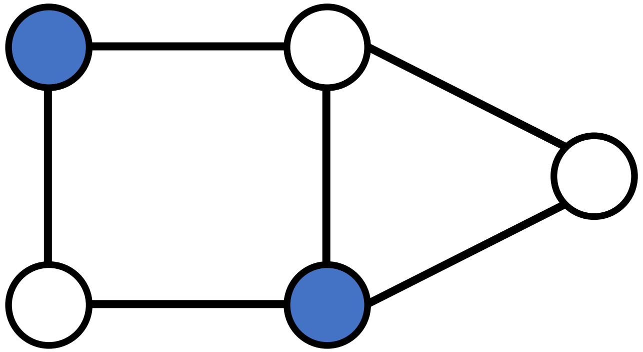
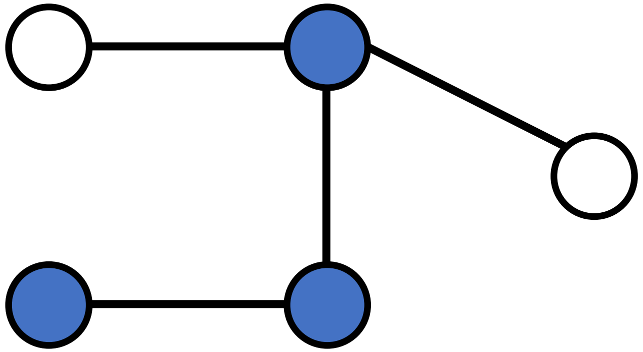


✗

Vertex Cover

Vertex Cover: Given ~~graph~~ tree, find the smallest subset of vertices such that every edge in the ~~graph~~ tree has at least one vertex in the subset.



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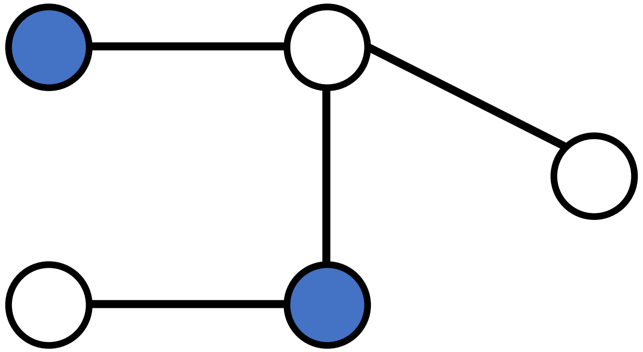
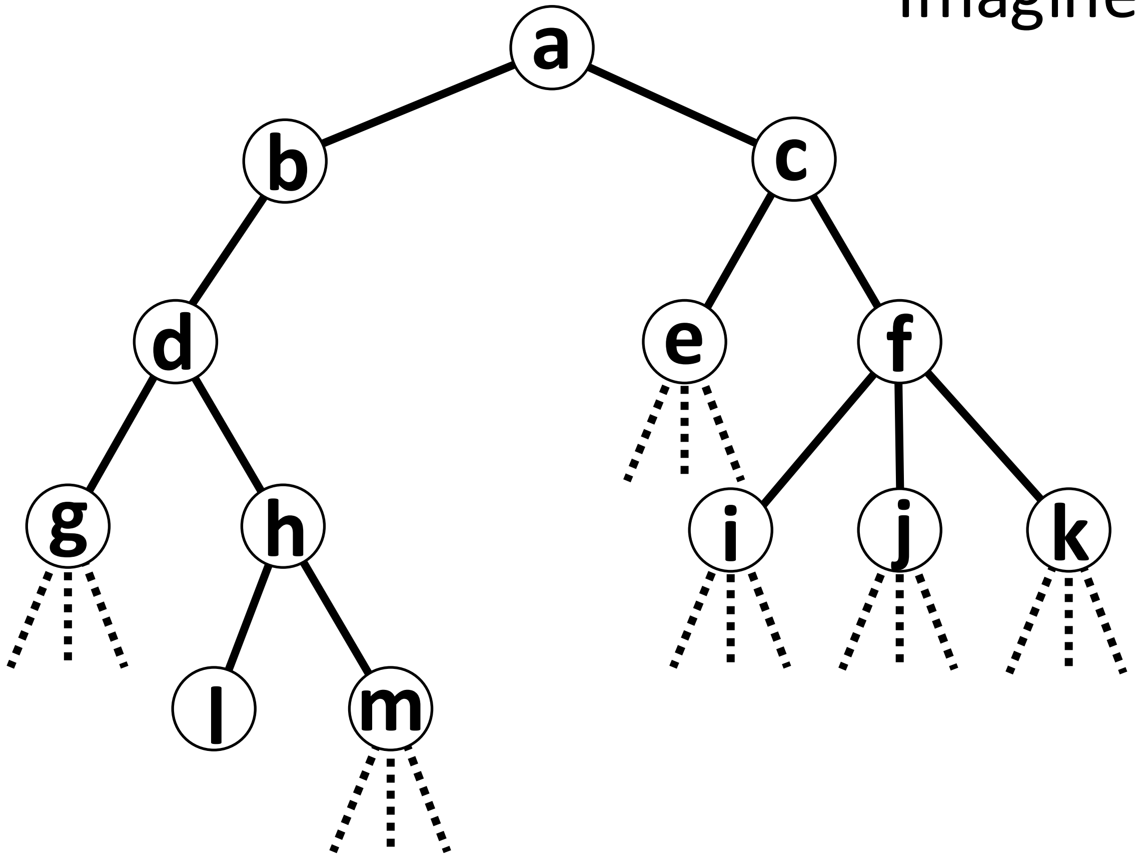


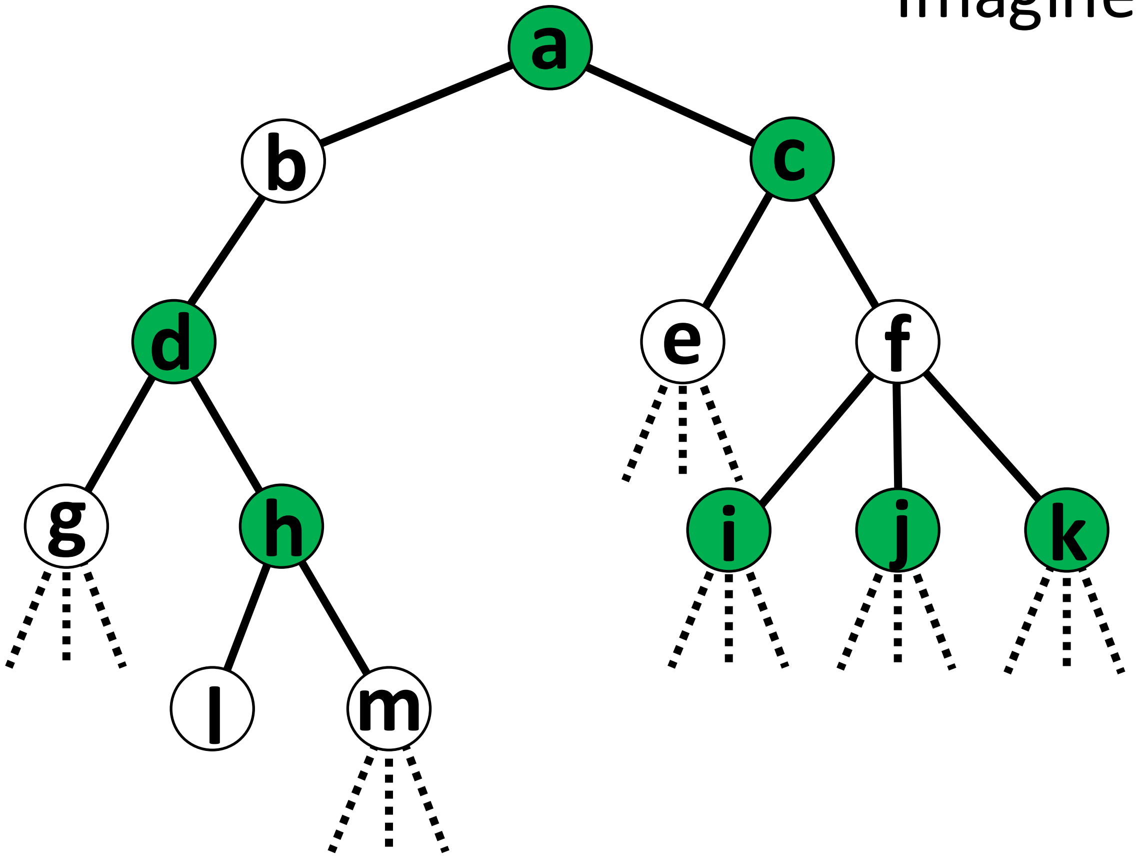
✗

Vertex Cover in Trees



Imagine the minimum vertex cover.

Vertex Cover in Trees



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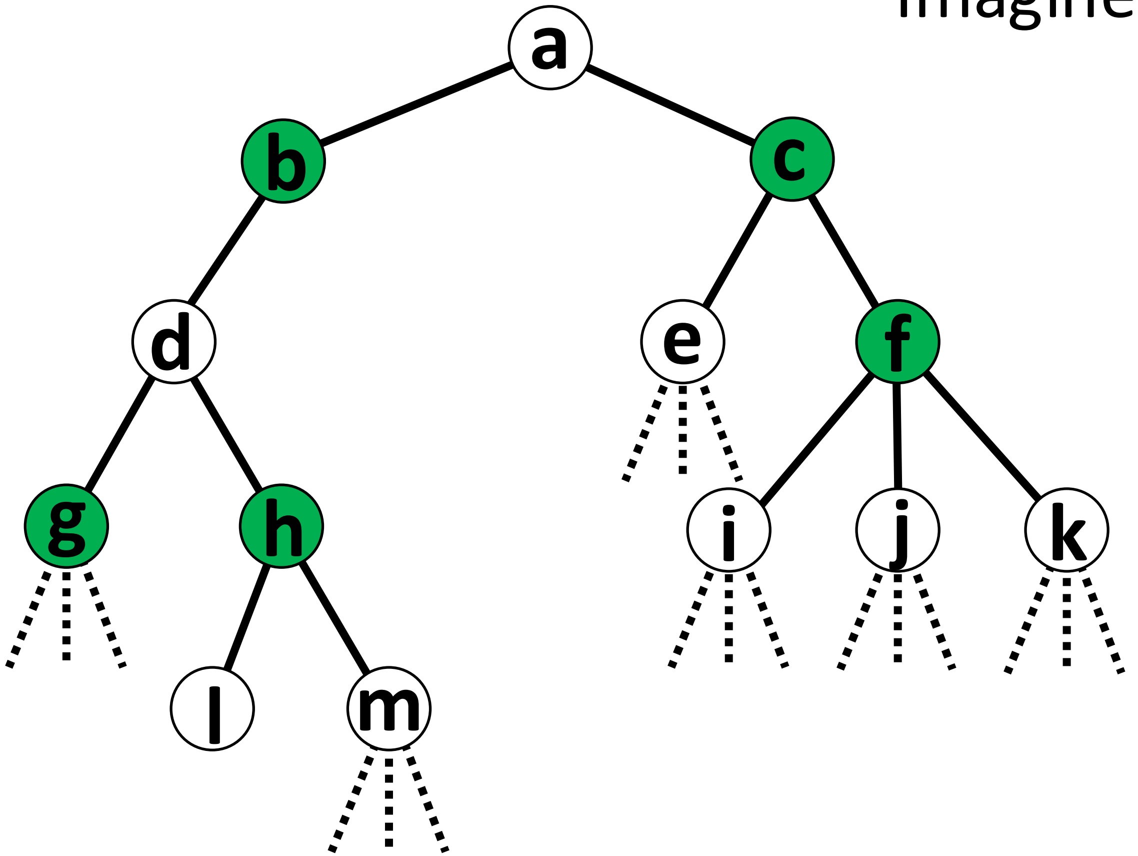
Vertex Cover in Trees



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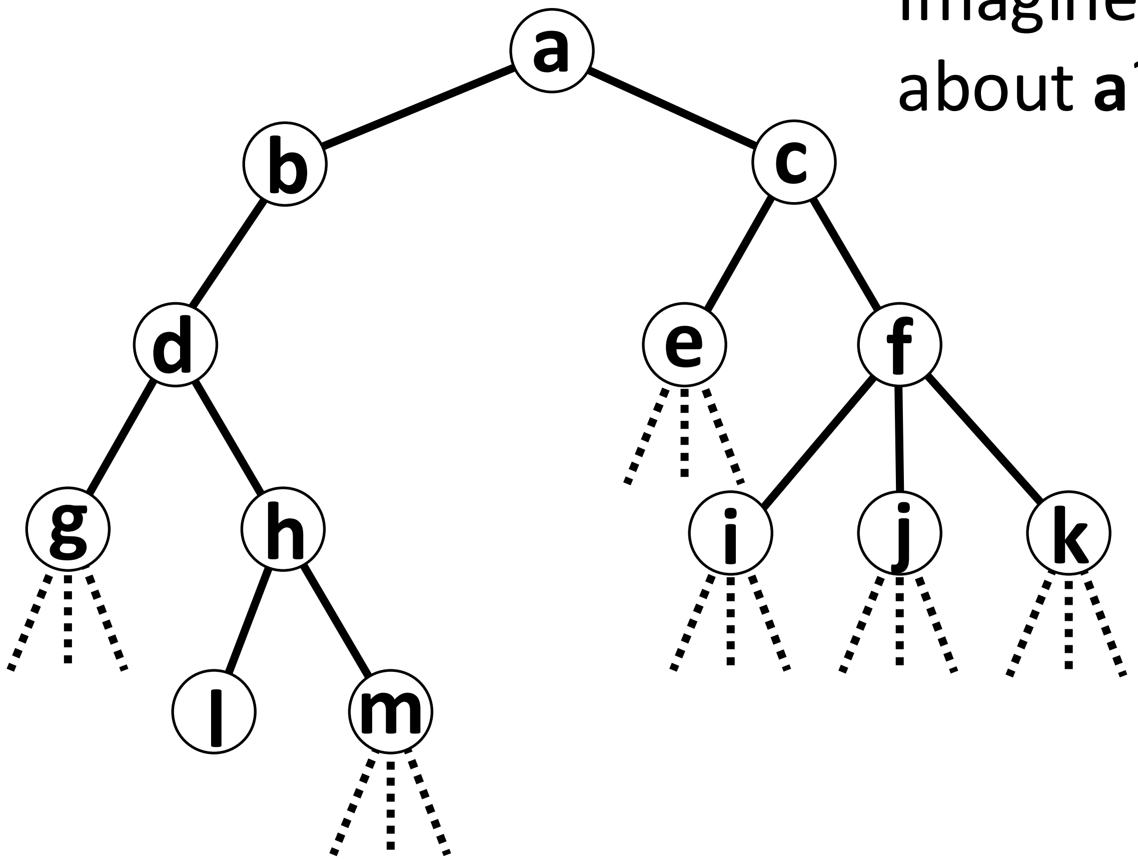
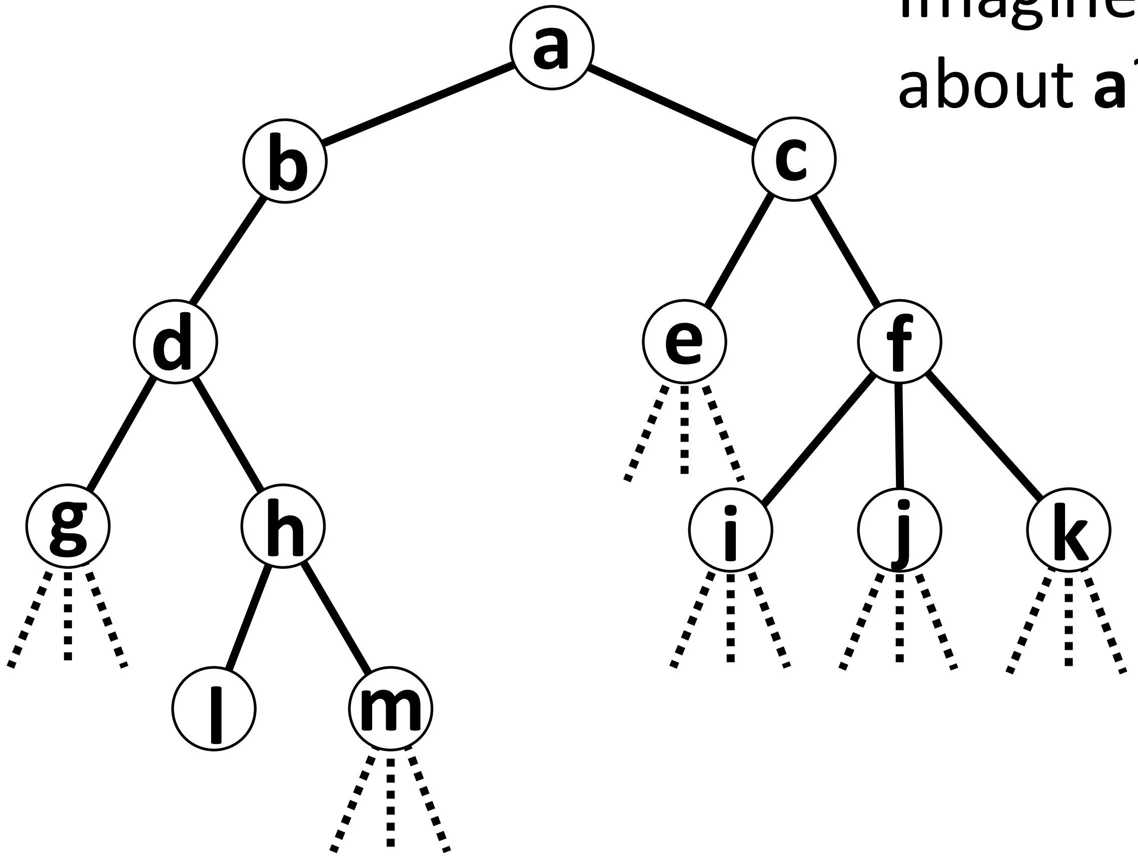
Vertex Cover in Trees



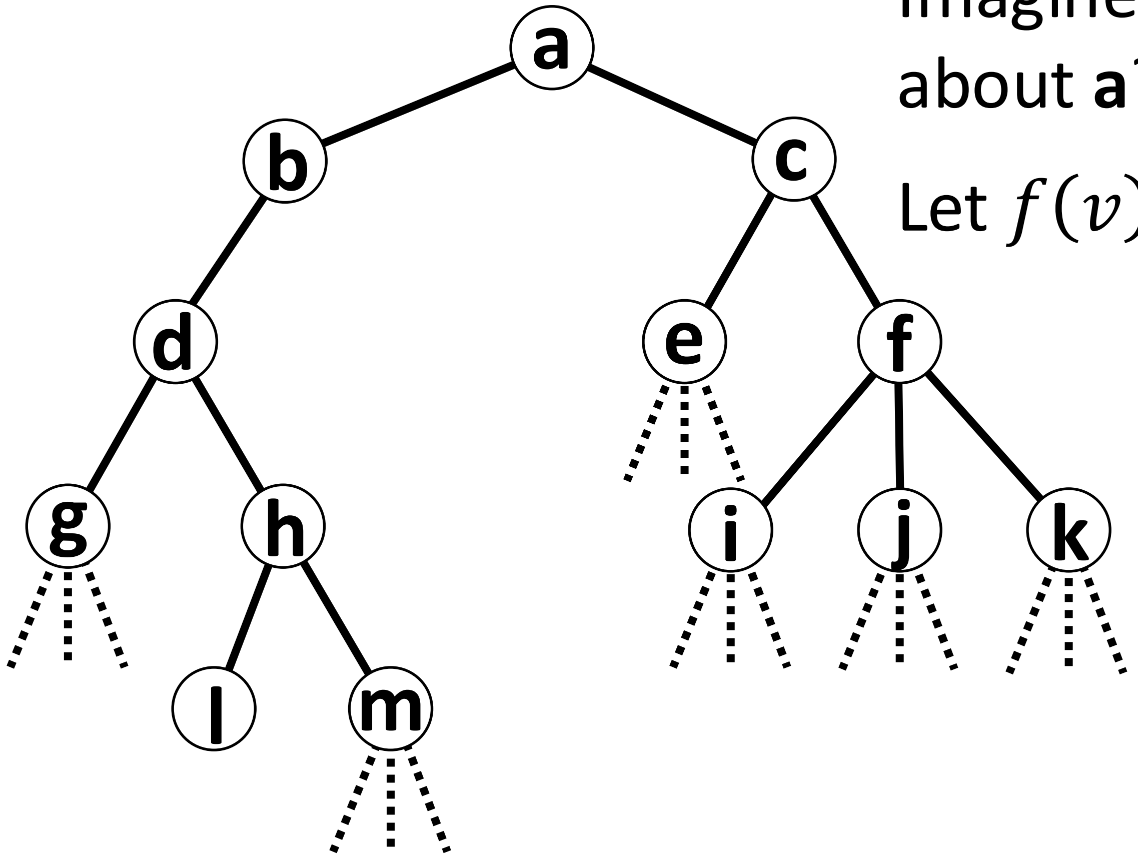
Imagine the minimum vertex cover. What can we say about **a**?

Vertex Cover in Trees



Imagine the minimum vertex cover. What can we say about **a**? **a** is in a minimum vertex cover, or it’s not.

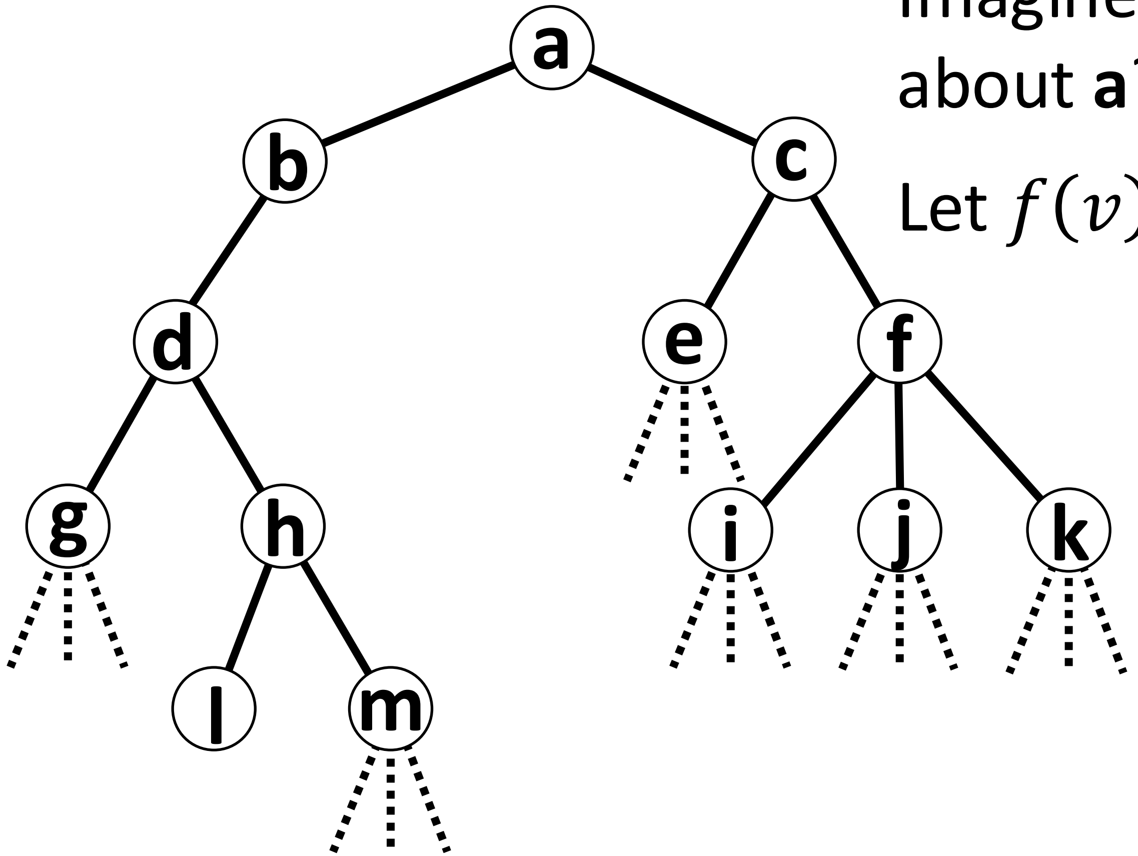
Vertex Cover in Trees



Imagine the minimum vertex cover. What can we say about **a**? **a** is in a minimum vertex cover, or it's not.

Let $f(v) =$ Size of minimum vertex cover rooted at v.

Vertex Cover in Trees



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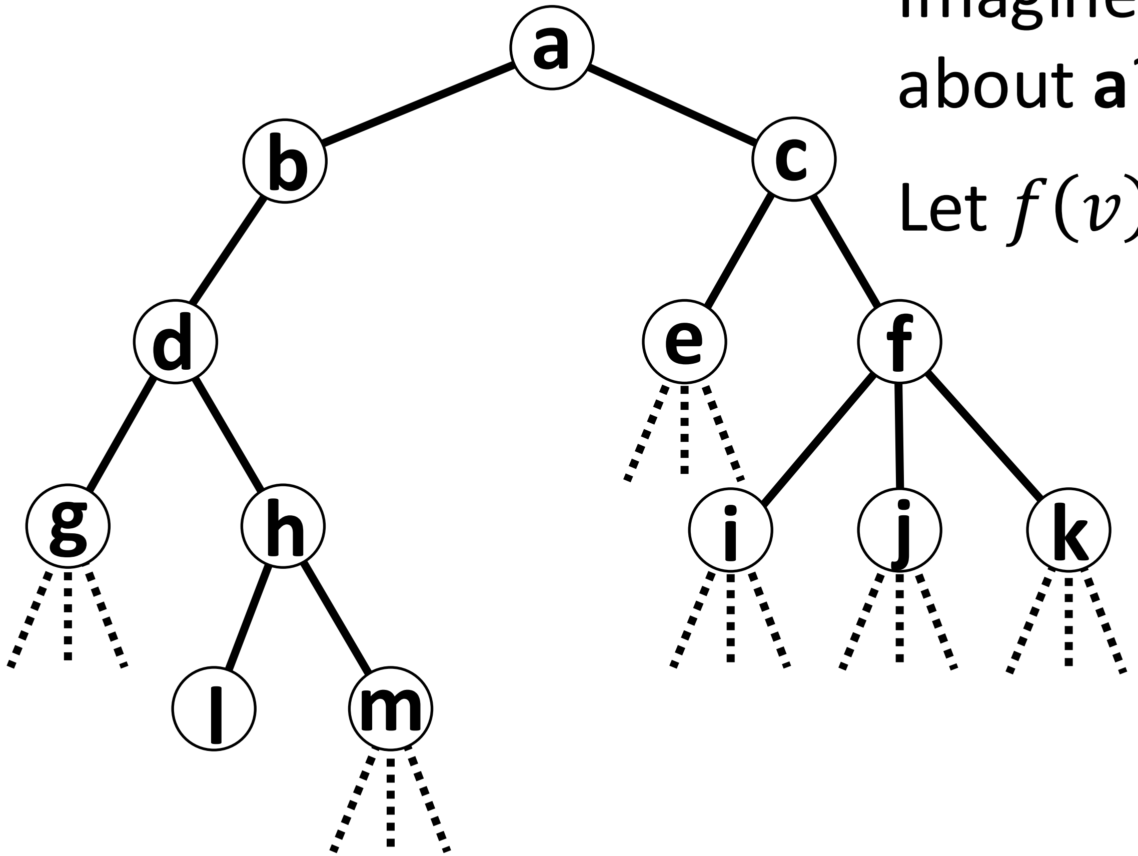
If a **<u>is</u>** in a minimum VC

If a **<u>is not</u>** in a minimum VC

Vertex Cover in Trees



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If a **is** in a minimum VC

$f(a) = ??$

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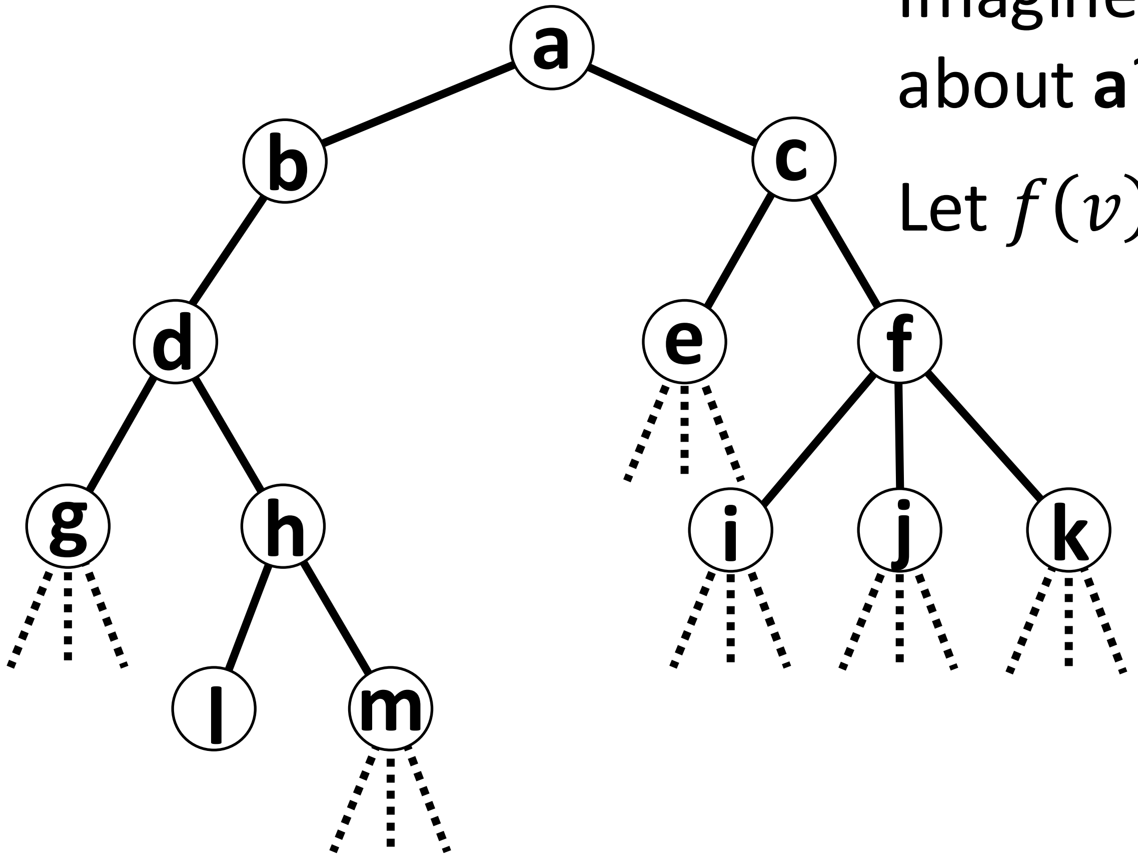

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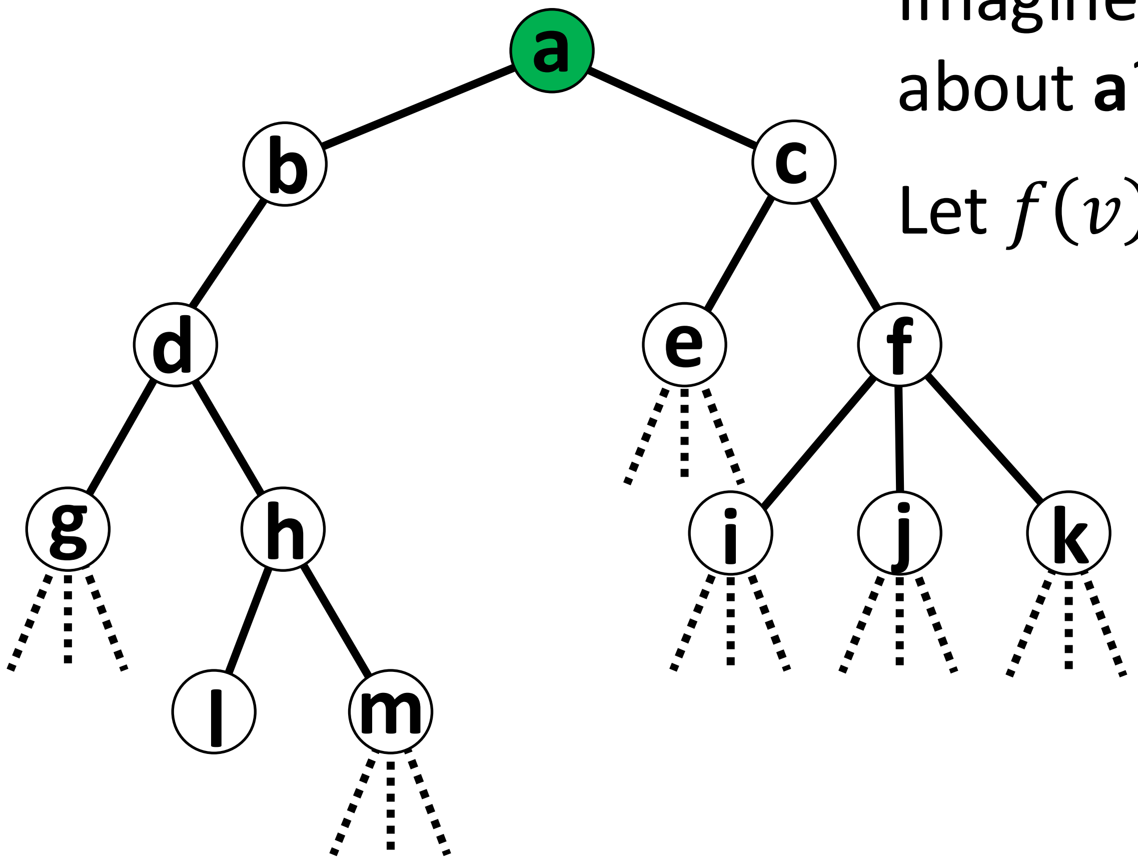
$f(a) =$??

Vertex Cover in Trees



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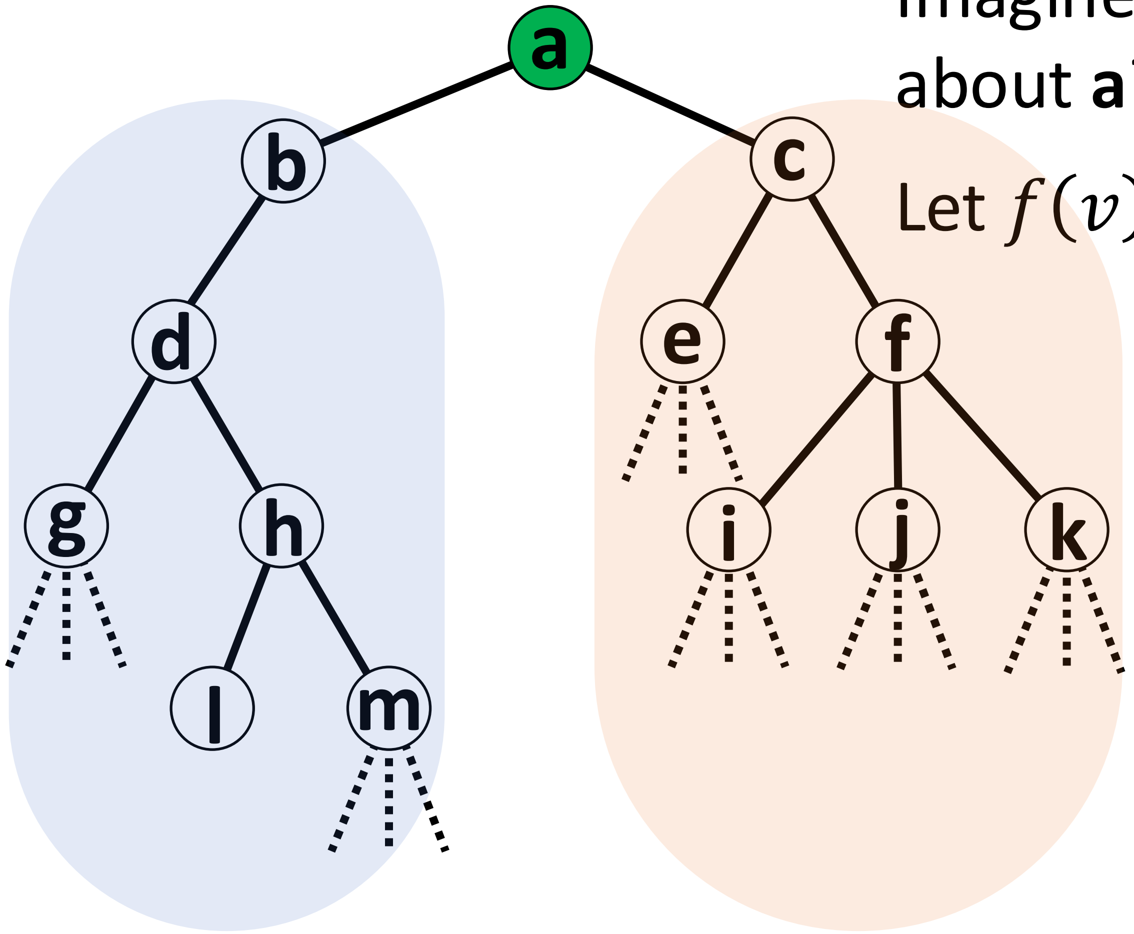
Let $f(v)$ = Size of minimum vertex cover rooted at v.

If a **is** in a minimum VC

$f(a) = 1$ + ??

Vertex Cover in Trees



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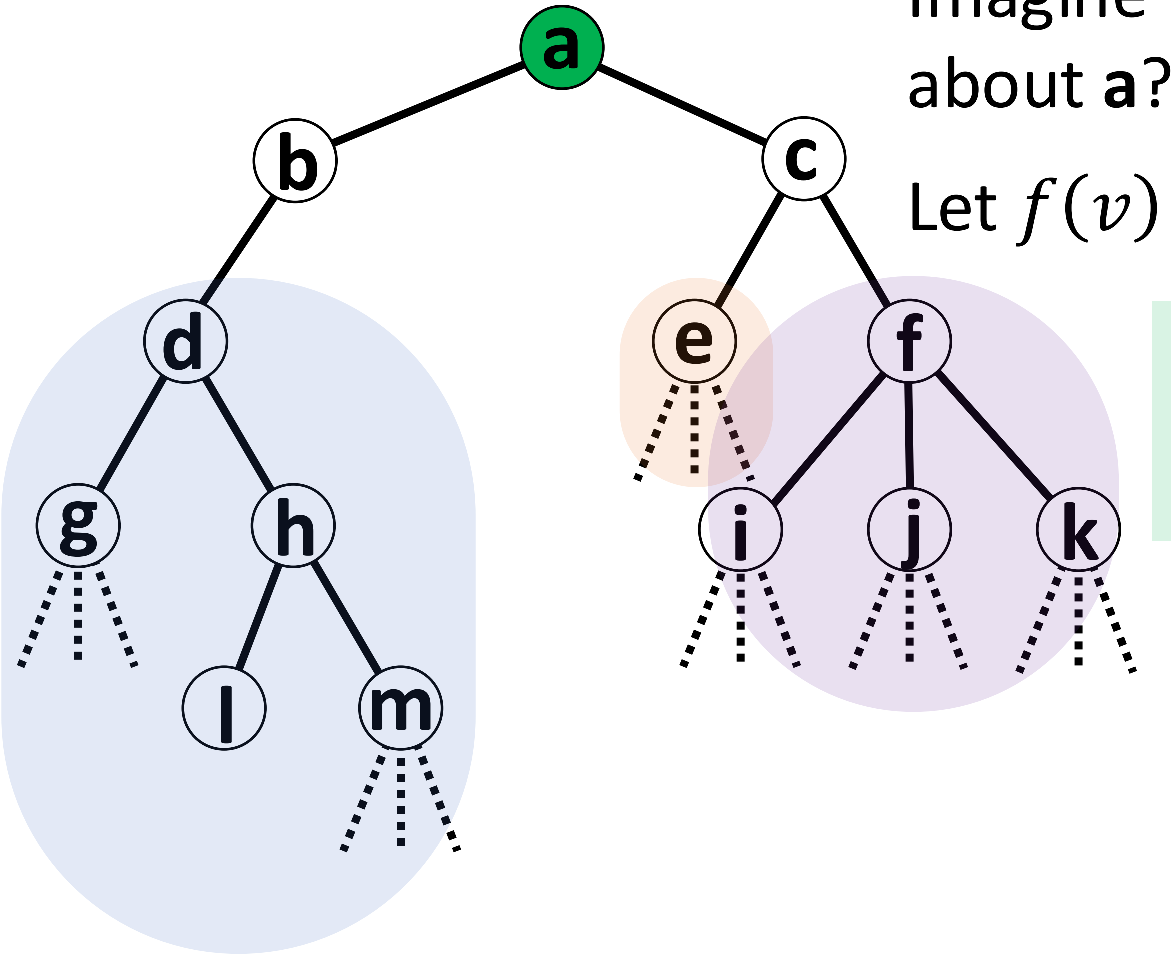
Let $f(v)$ = Size of minimum vertex cover rooted at v.

If a <u>**is**</u> in a minimum VC

$$f(a) = 1 + f(b) + f(c)$$

Vertex Cover in Trees



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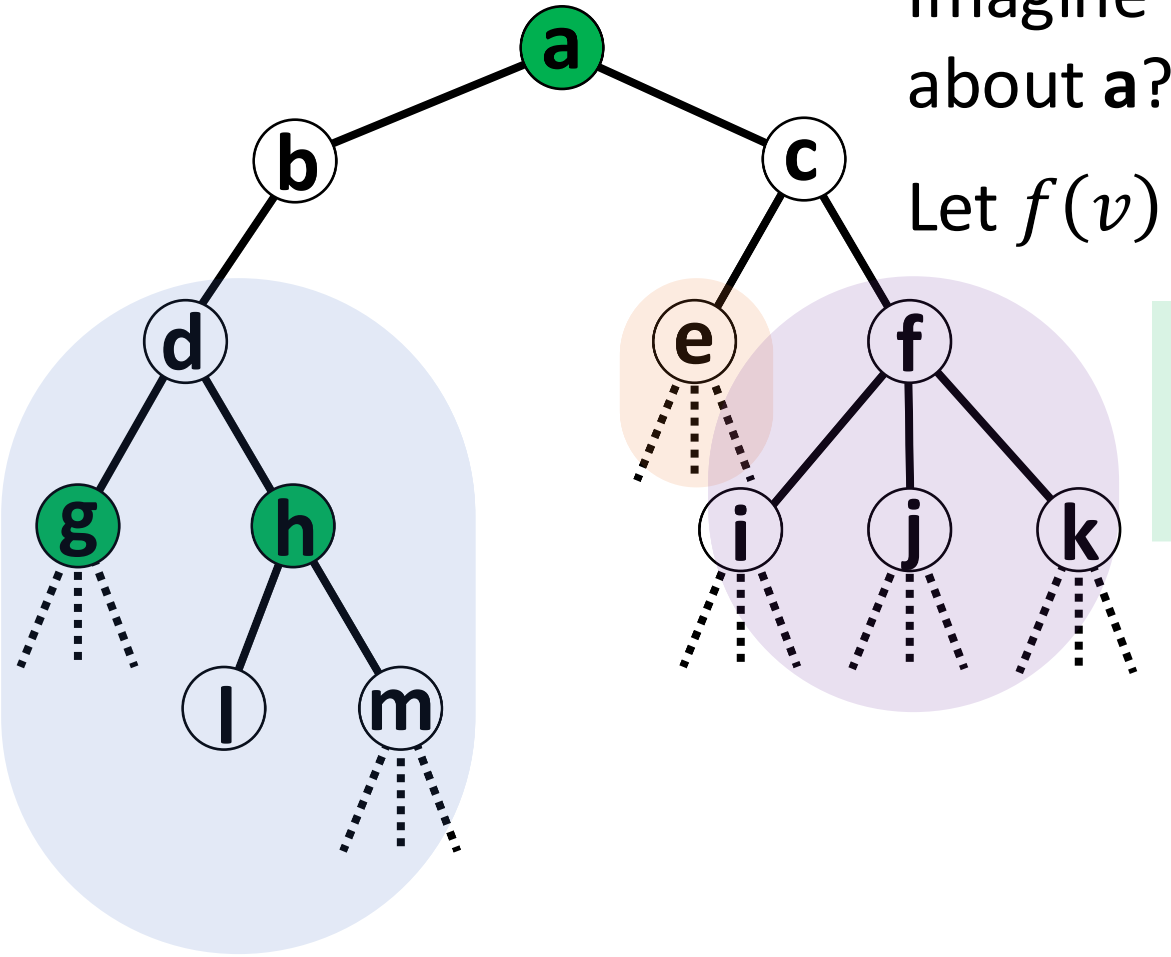
Let $f(v)$ = Size of minimum vertex cover rooted at v.

If a **is** in a minimum VC

$$f(a) = 1 + f(d) + f(e) + f(f)$$

Why not this?

Vertex Cover in Trees



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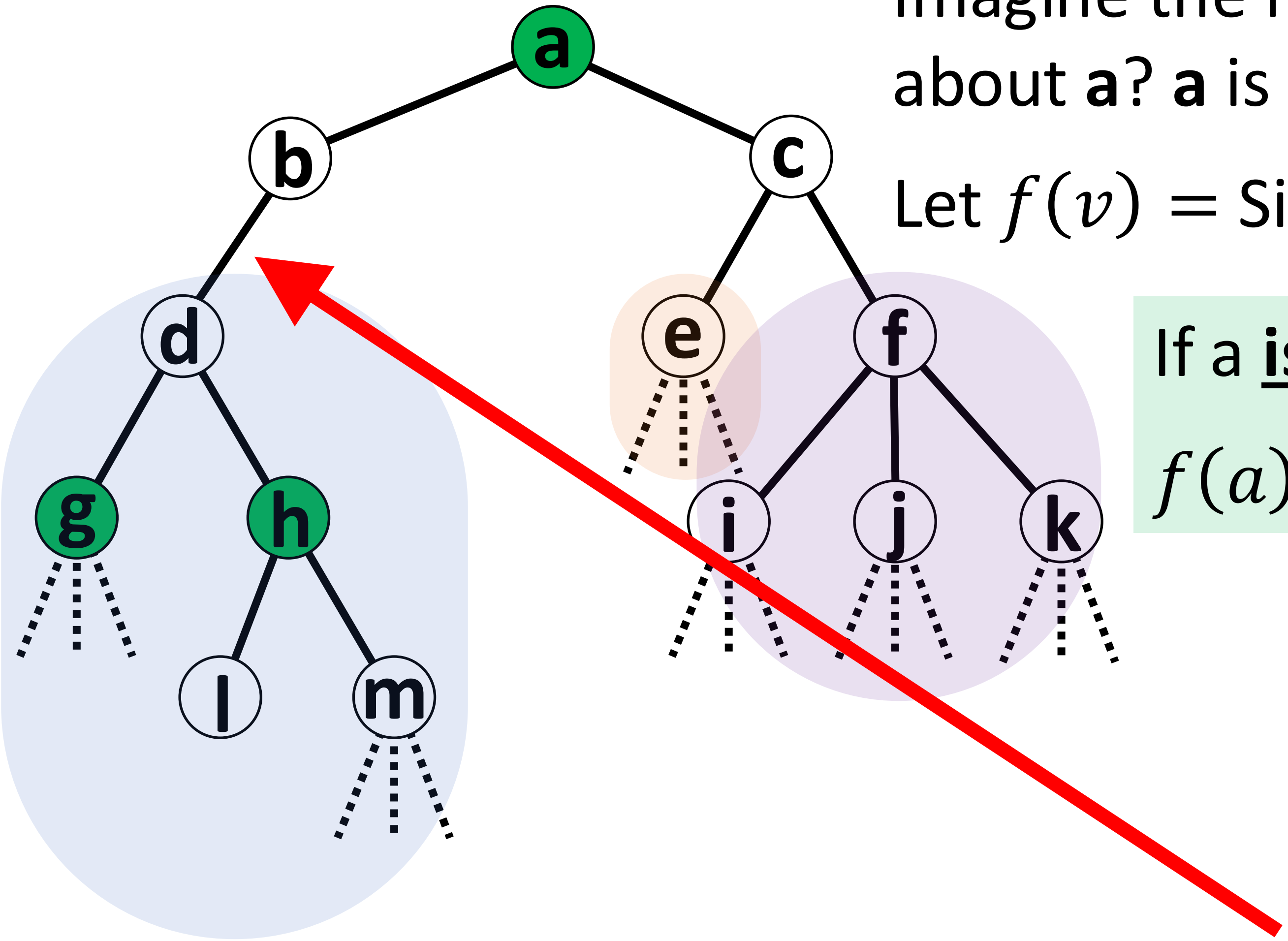
$$f(a) = 1 + f(d) + f(e) + f(f)$$

Why not this?

Because the optimal VC rooted at **d** may not include **d**…

Vertex Cover in Trees



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Let $f(v)$ = Size of minimum vertex cover rooted at v.

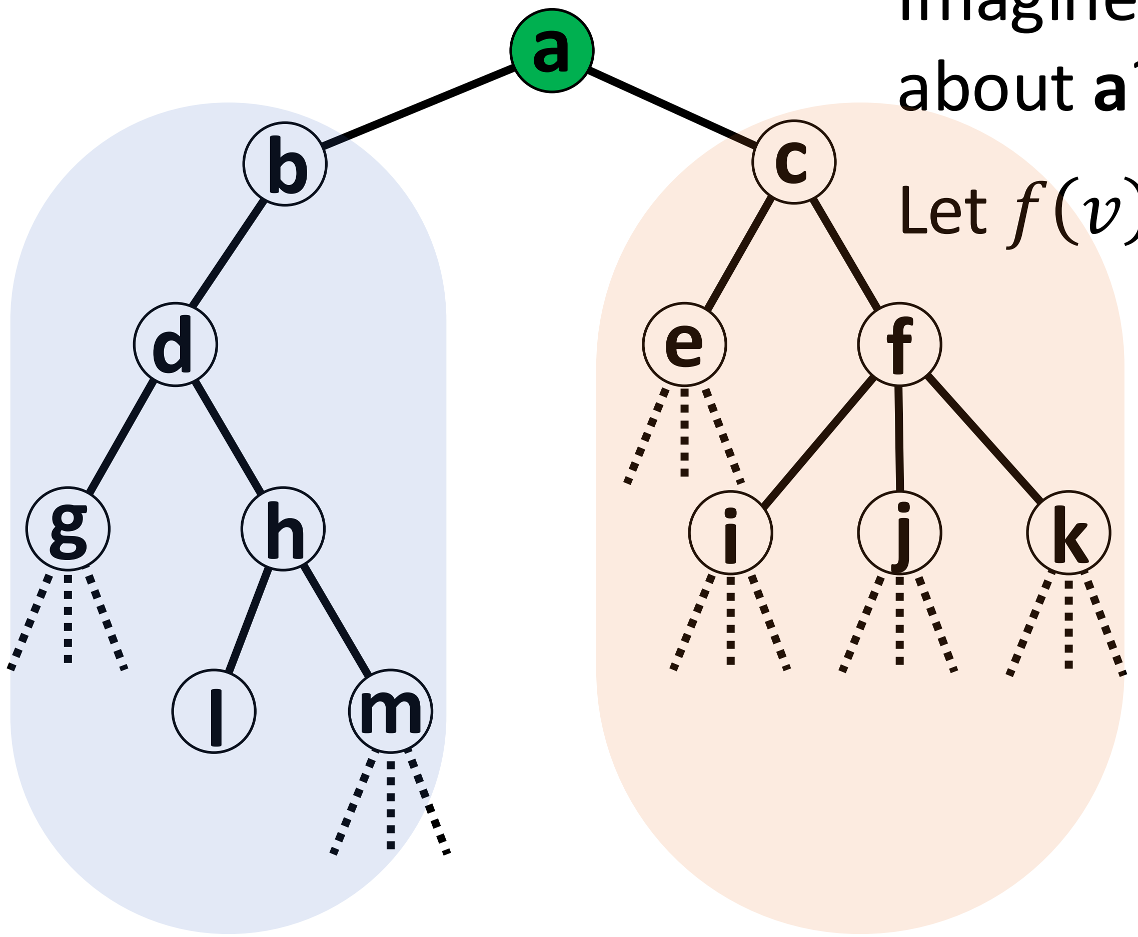
If a <u>**is**</u> in a minimum VC

$$f(a) = 1 + f(d) + f(e) + f(f)$$

Why not this?

Because the optimal VC rooted at **d** may not include **d**…which leaves an edge uncovered.

Vertex Cover in Trees



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Let $f(v) =$ Size of minimum vertex cover rooted at v.

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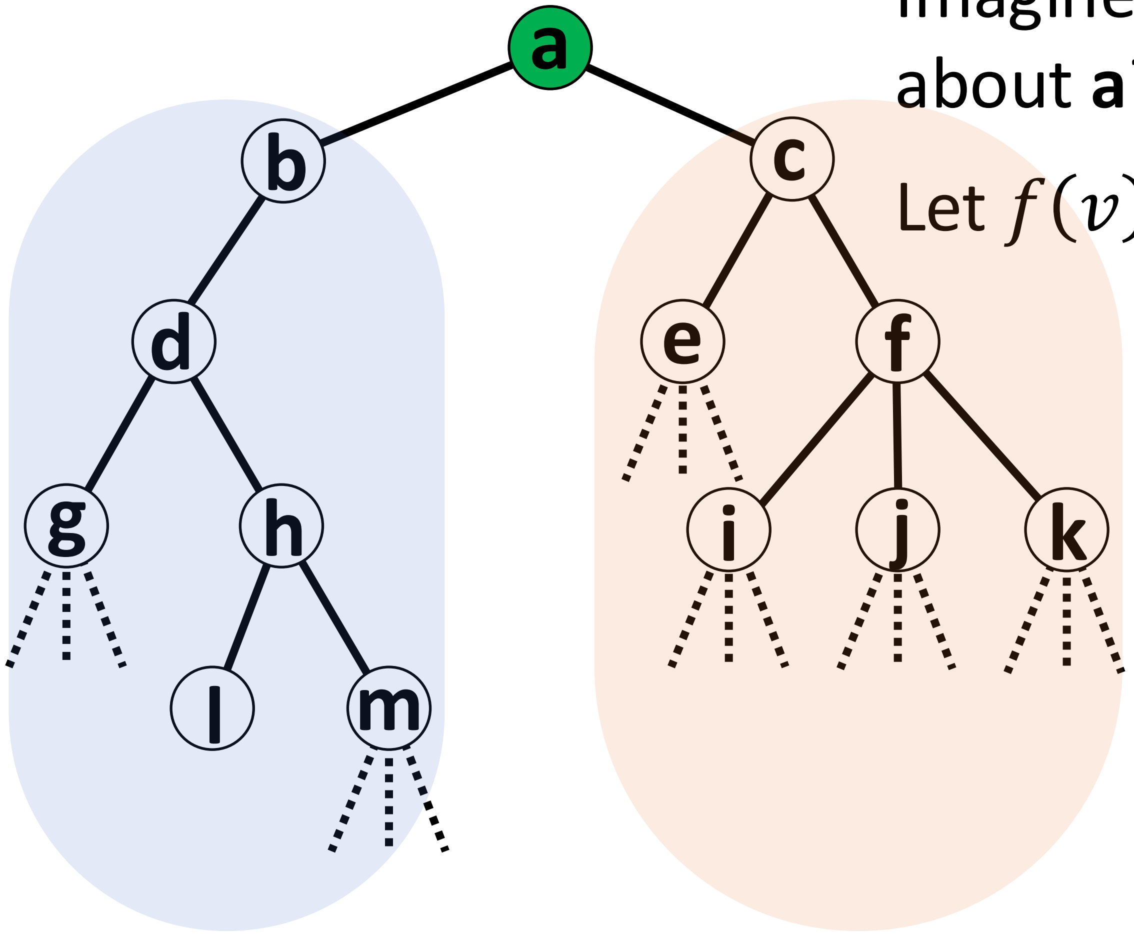
$$f(a) = 1 + f(b) + f(c)$$

Vertex Cover in Trees



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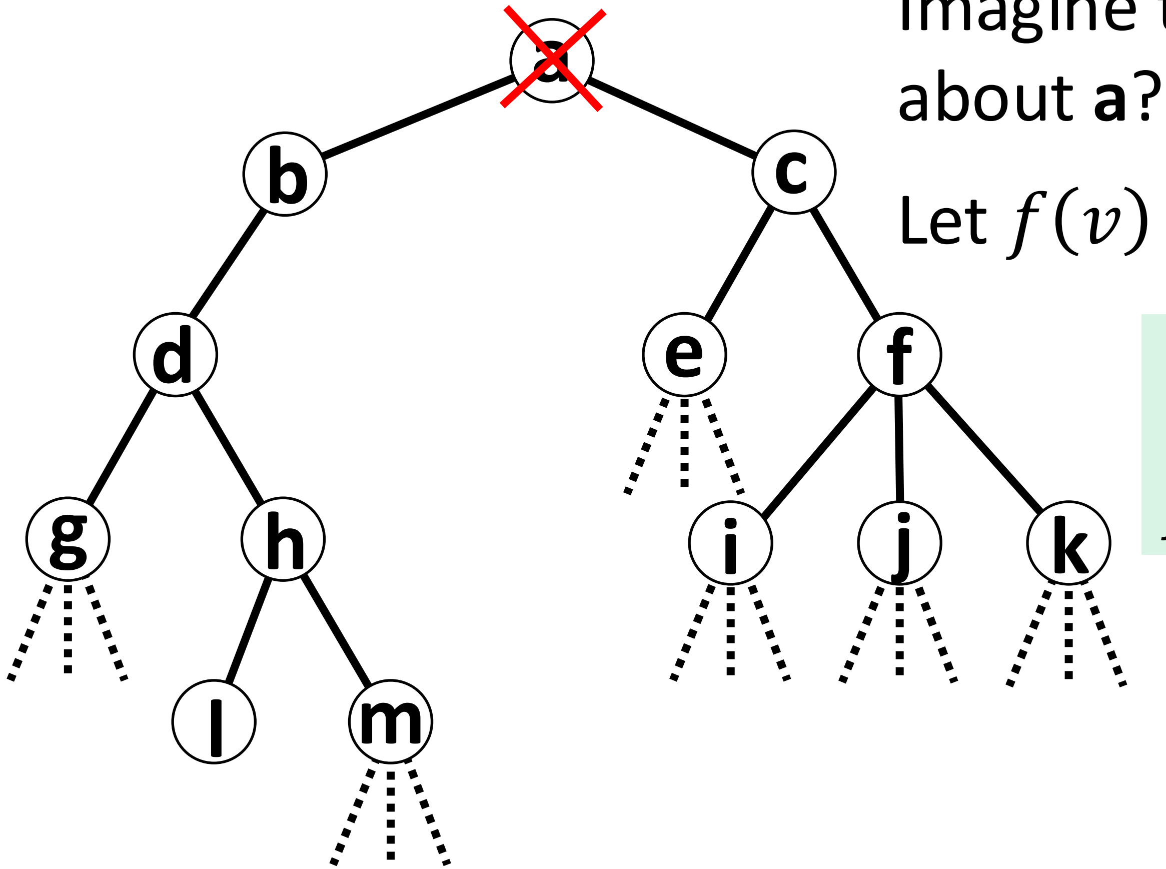
Let $f(v)$ = Size of minimum vertex cover rooted at v.

If a **is** in a minimum VC

$$f(a) = 1 + f(b) + f(c)$$

*Optimal Substructure: "If there was a smaller VC rooted at **b**, it would give us a smaller VC rooted at **a**."*

Vertex Cover in Trees



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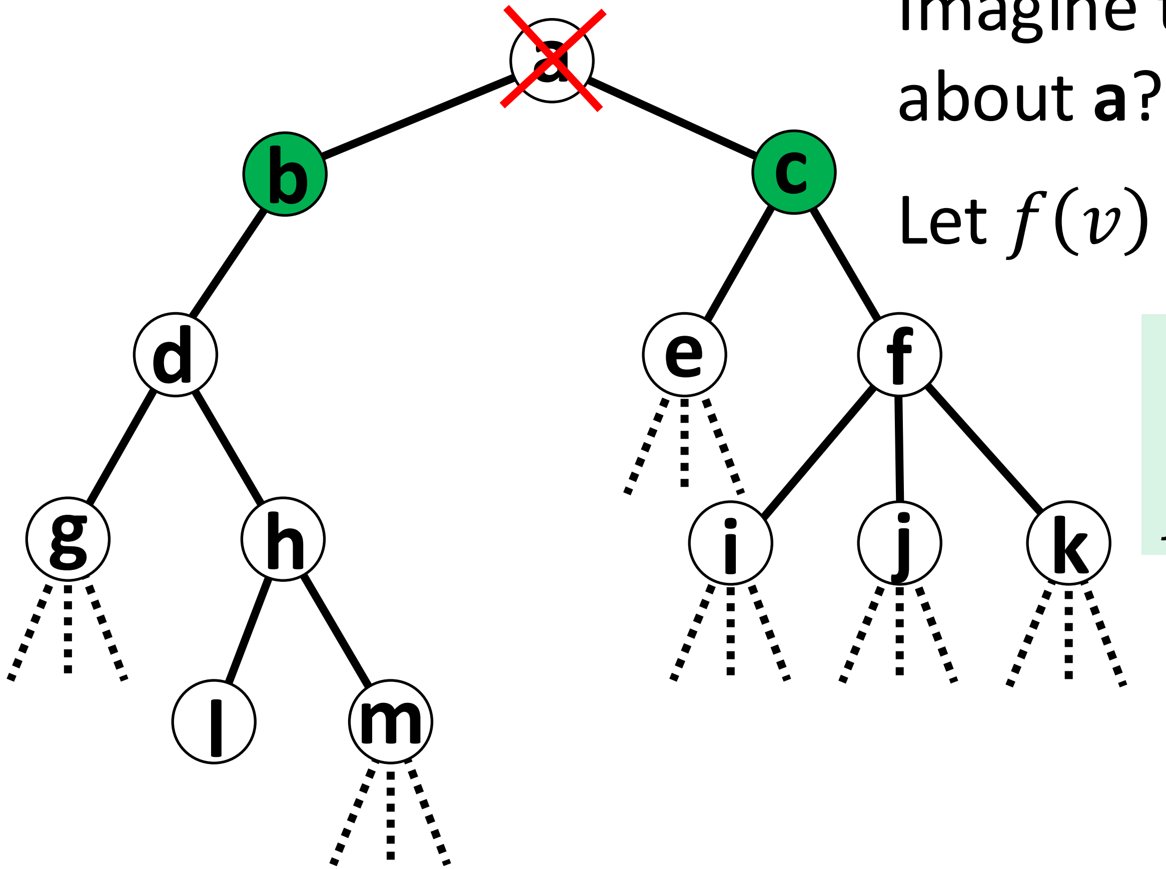
If a **<u>is</u>** in a minimum VC

$$f(a) = 1 + f(b) + f(c)$$

If a **<u>is not</u>** in a minimum VC

$$f(a) = ??$$

Vertex Cover in Trees



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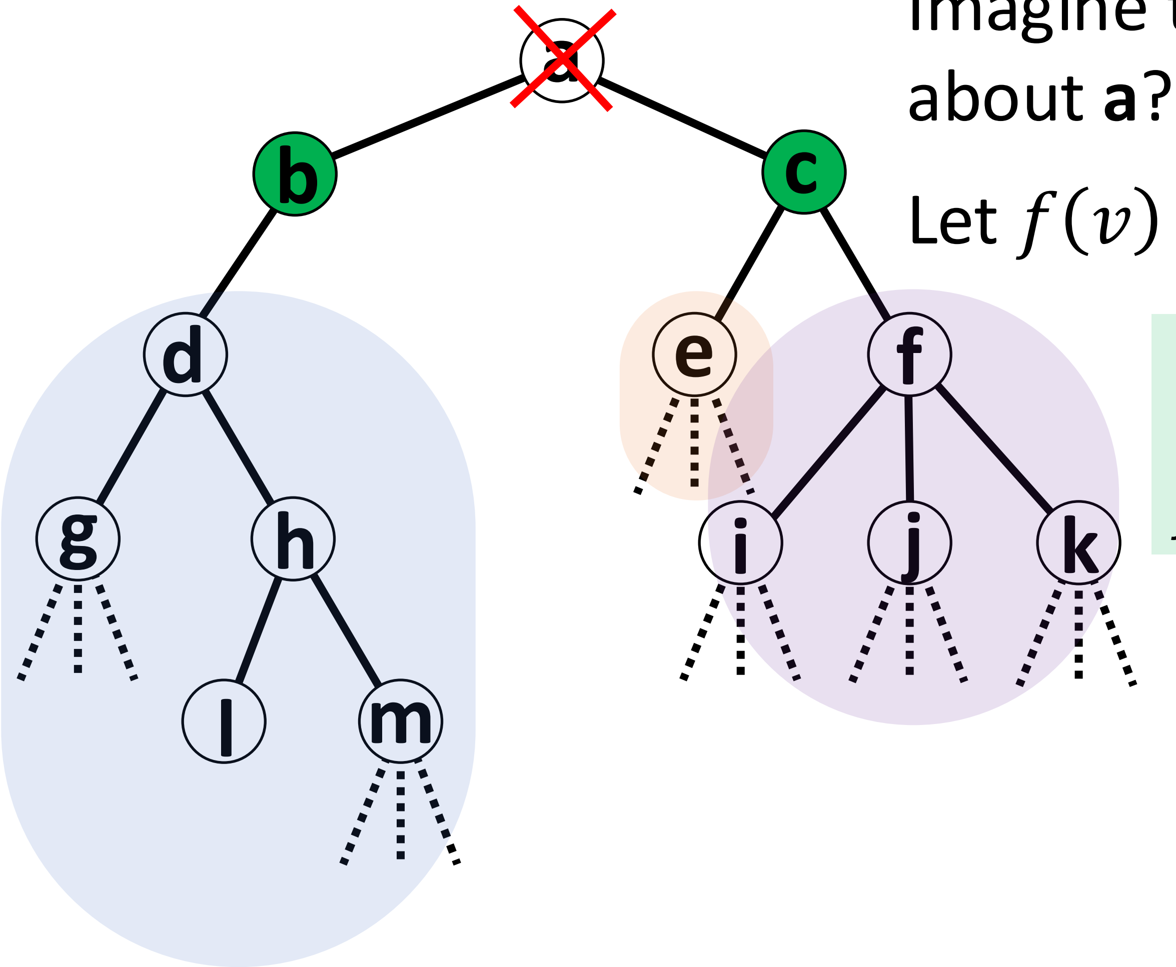
If a **<u>is</u>** in a minimum VC

$f(a) = 1 + f(b) + f(c)$

If a **<u>is not</u>** in a minimum VC

$f(a) = 2 + ??$

Vertex Cover in Trees



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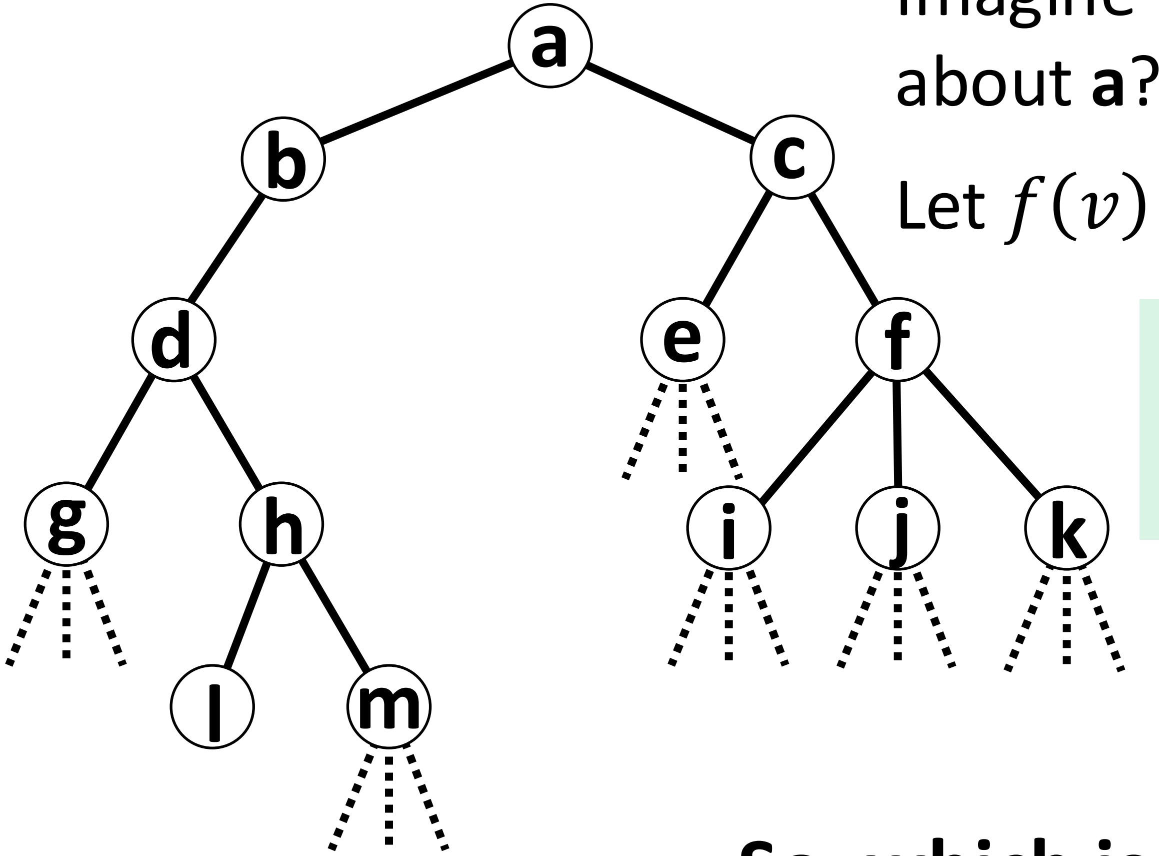
$$f(a) = 1 + f(b) + f(c)$$

If a **<u>is not</u>** in a minimum VC

$$f(a) = 2 + f(d) + f(e) + f(f)$$

Vertex Cover in Trees



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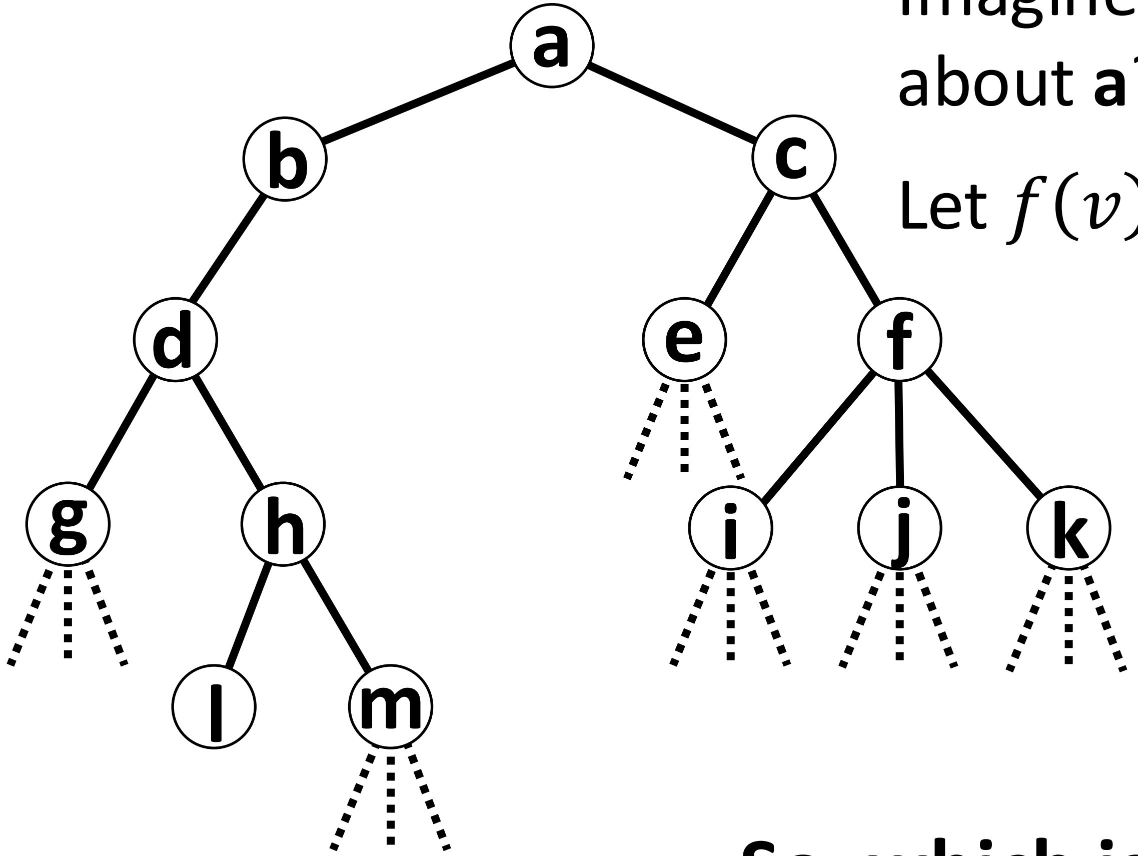
$$f(a) = 1 + f(b) + f(c)$$

If a **<u>is not</u>** in a minimum VC

$$f(a) = 2 + f(d) + f(e) + f(f)$$

So, which is it?

Vertex Cover in Trees



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If a **is** in a minimum VC

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If a **is not** in a minimum VC

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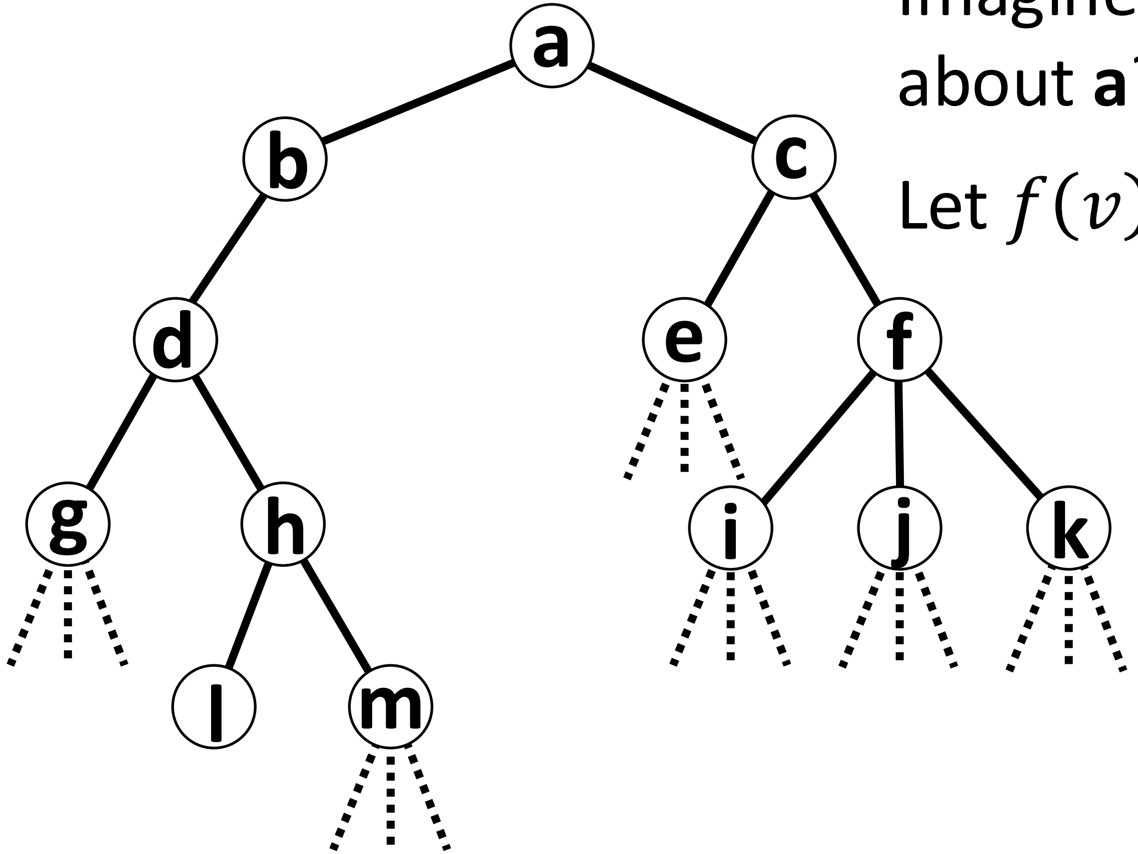
So, which is it?

The smallest!

Vertex Cover in Trees



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If a **<u>is</u>** in a minimum VC

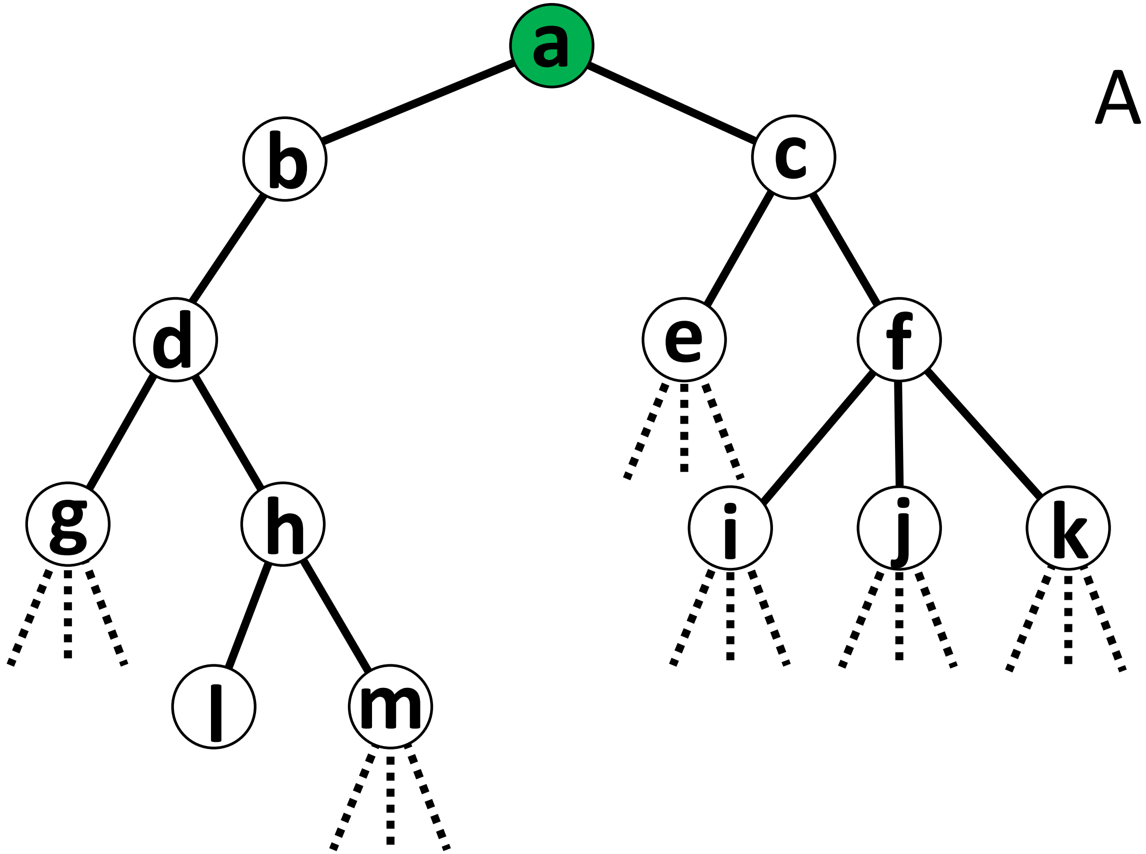
$$f(a) = 1 + f(b) + f(c)$$

If a **<u>is not</u>** in a minimum VC

$$f(a) = 2 + f(d) + f(e) + f(f)$$

$$f(v) = \min \left\{ 1 + \sum_{u \in \text{child}(v)} f(u),\ |\text{child}(v)| + \sum_{w \in \text{grandchild}(v)} f(w) \right\}$$

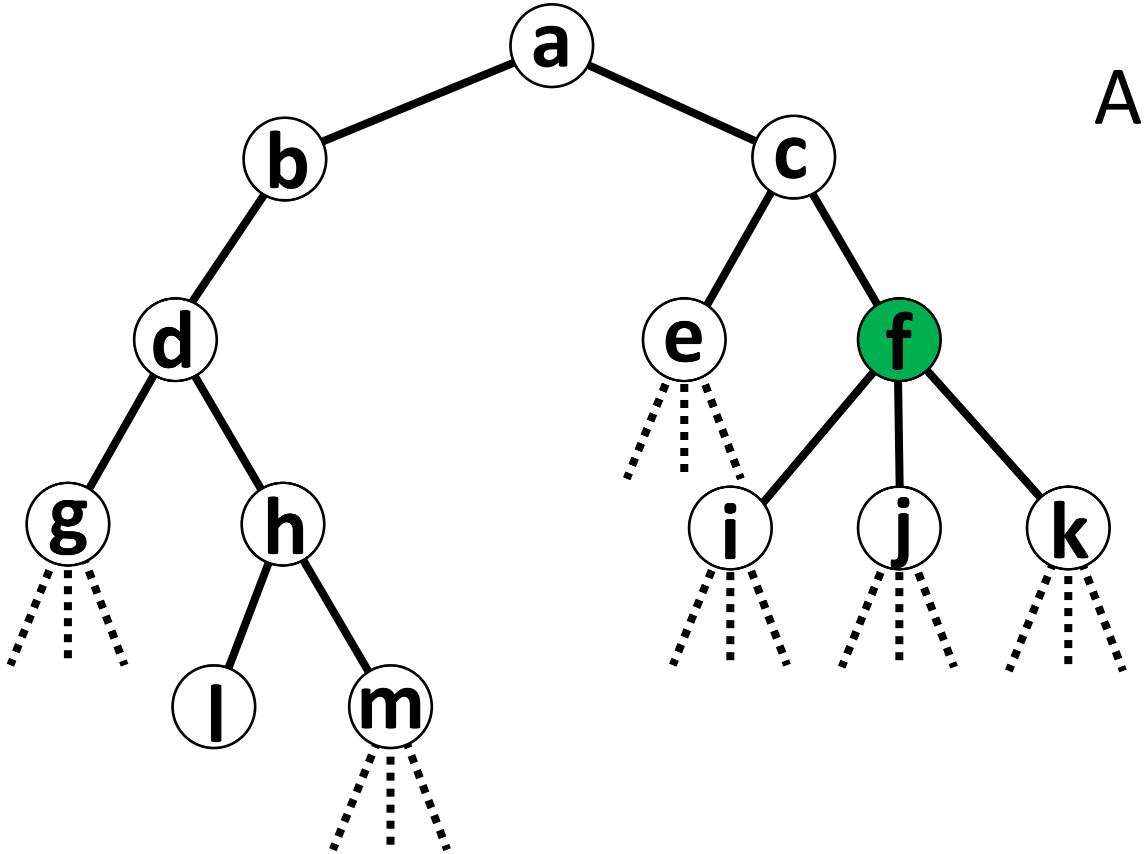
Vertex Cover in Trees



Are we ready to calculate $f(v)$ **here**?

$$f(v) = \min\left\{1 + \sum_{u \in \text{child}(v)} f(u)\,,\ |\text{child}(v)| + \sum_{w \in \text{grandchild}(v)} f(w)\right\}$$

Vertex Cover in Trees



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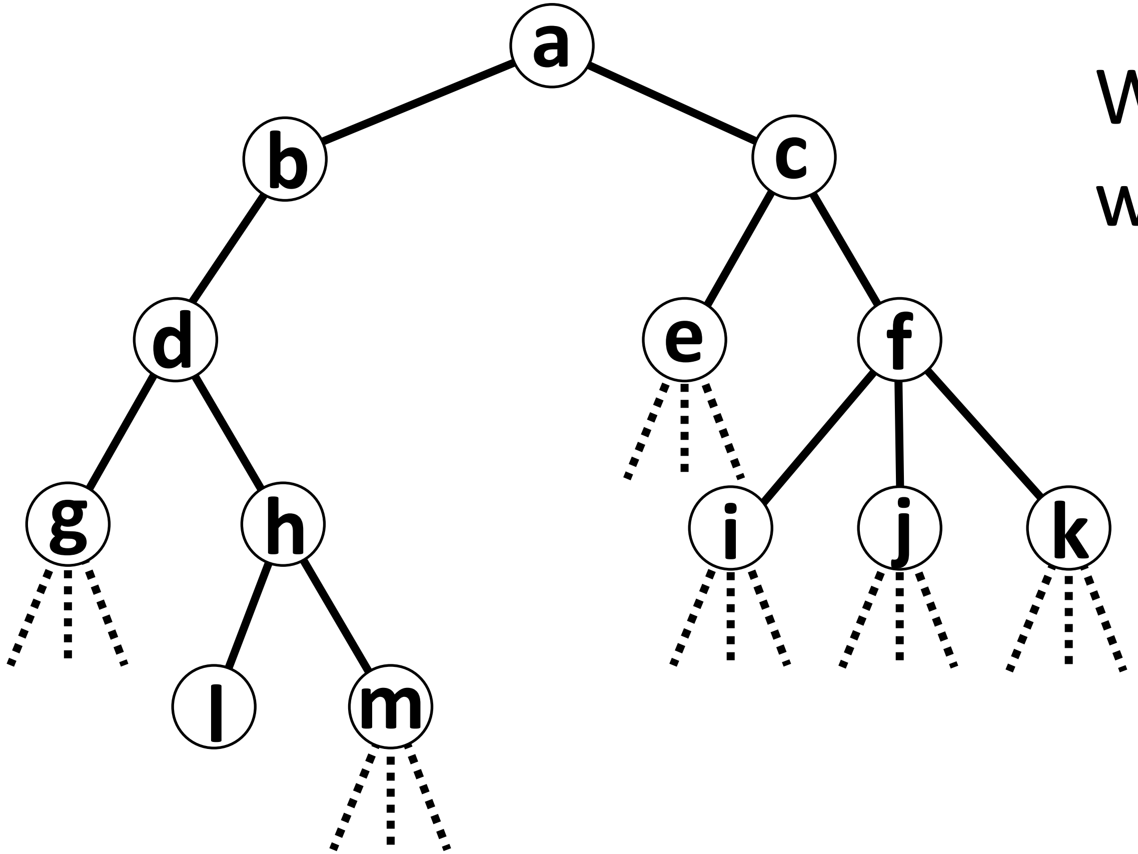
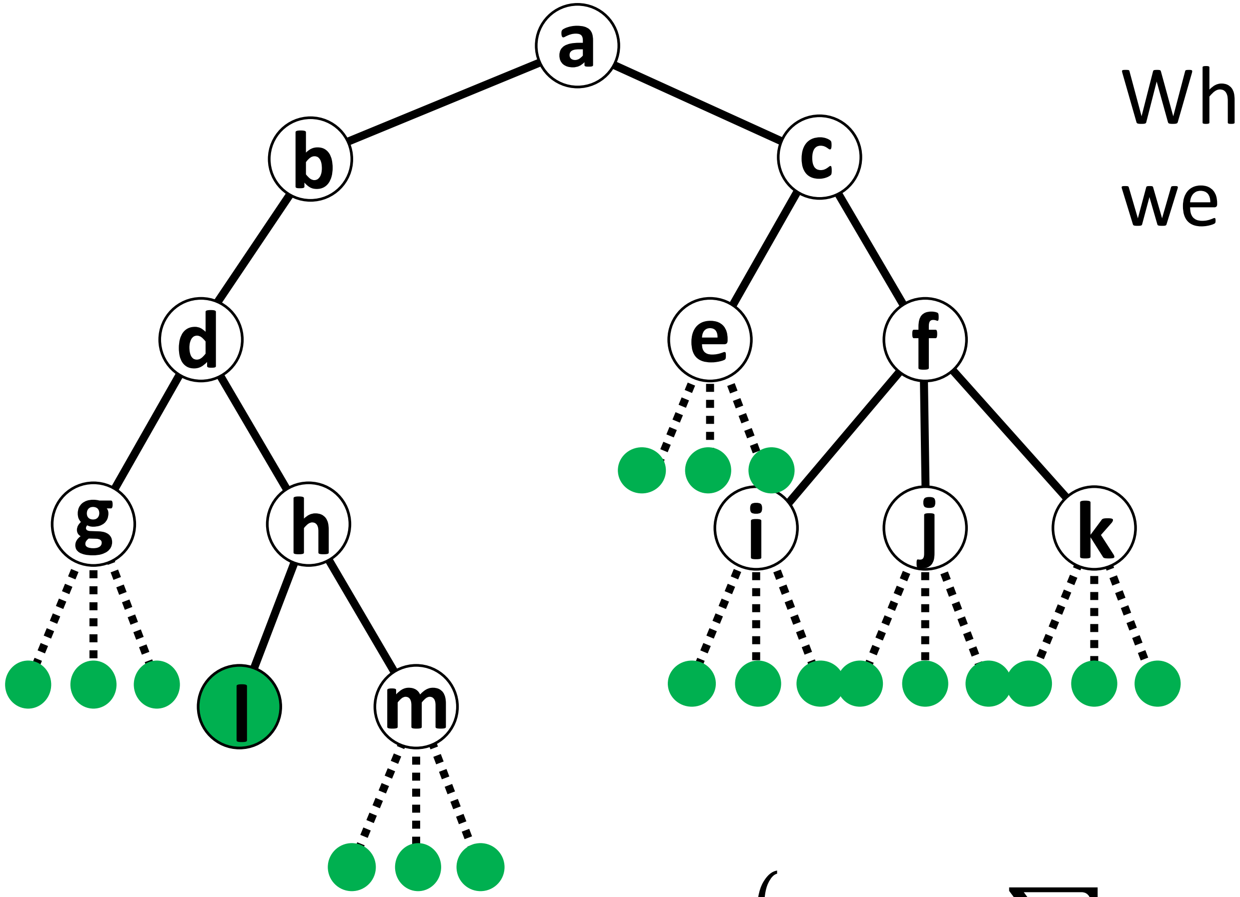


Where do we have all the information we need to calculate $f(v)$?

$$f(v) = \min\left\{1 + \sum_{u \in \text{child}(v)} f(u)\,,\ |\text{child}(v)| + \sum_{w \in \text{grandchild}(v)} f(w)\right\}$$

Vertex Cover in Trees



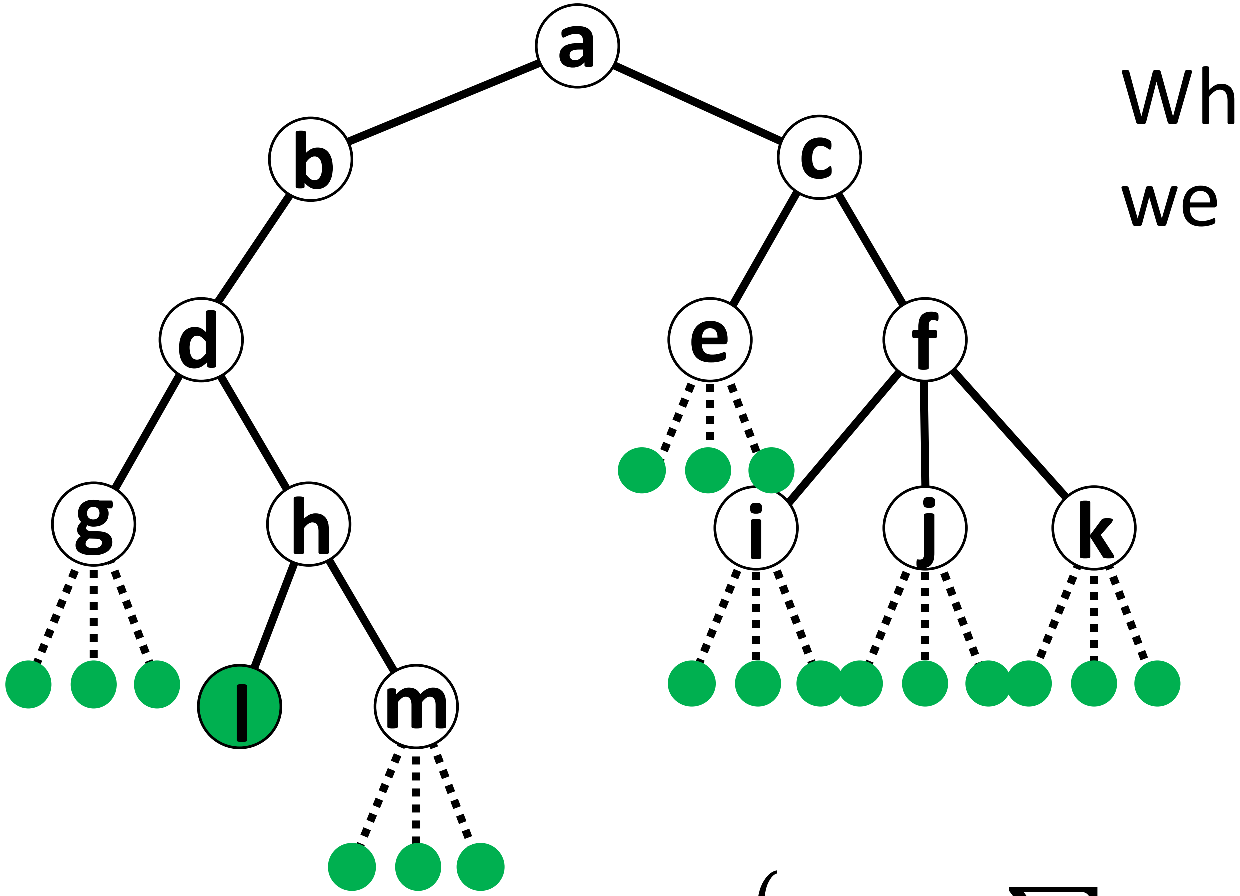
Where do we have all the information we need to calculate $f(v)$?

At the leaves!!

$$f(v) = \min\left\{1 + \sum_{u \in \text{child}(v)} f(u)\,,\ |\text{child}(v)| + \sum_{w \in \text{grandchild}(v)} f(w)\right\}$$

Vertex Cover in Trees

Let $f(v)$ = Size of minimum vertex cover rooted at v.



Where do we have all the information we need to calculate $f(v)$?

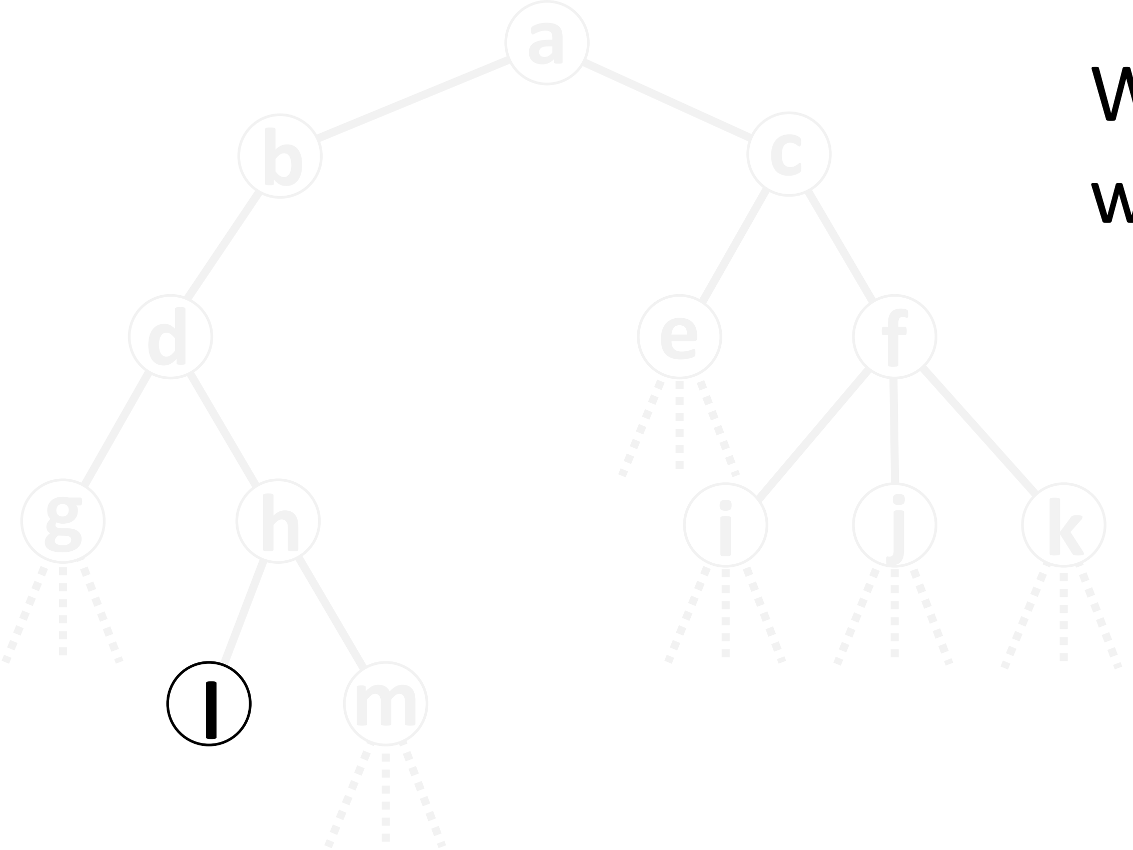
At the leaves!!

If v is a leaf, $f(v) = ??$

$$f(v) = \min\left\{1 + \sum_{u \in \text{child}(v)} f(u),\ |\text{child}(v)| + \sum_{w \in \text{grandchild}(v)} f(w)\right\}$$

Vertex Cover in Trees

Let $f(v)$ = Size of minimum vertex cover rooted at v.



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Vertex Cover in Trees

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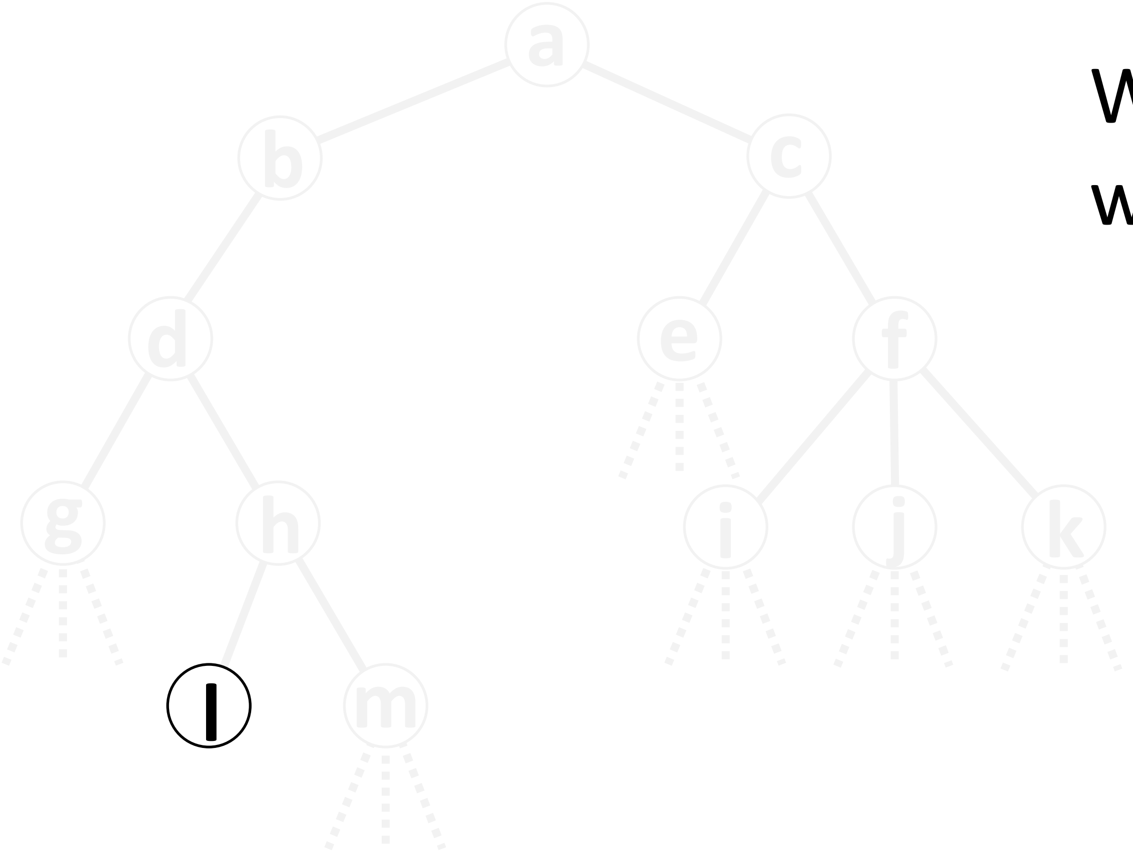


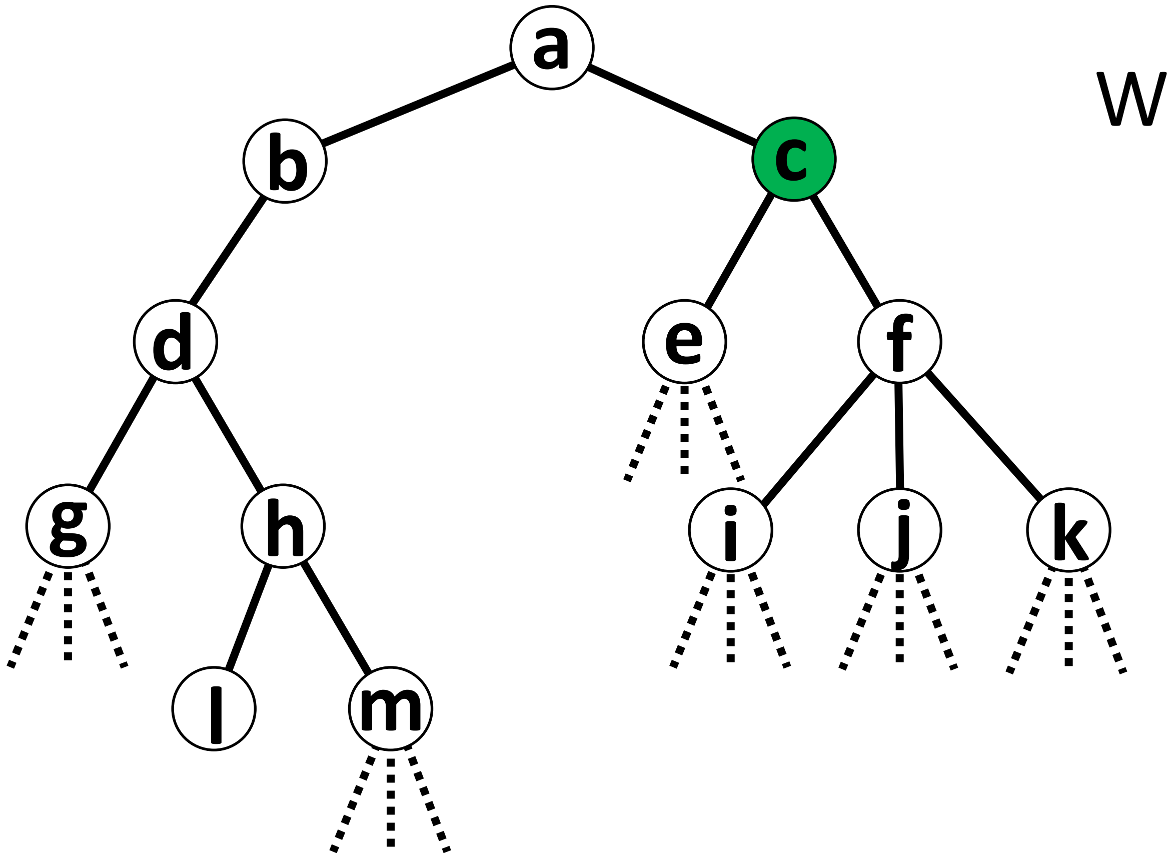
Where do we have all the information we need to calculate $f(v)$?

At the leaves!!

If v is a leaf, $f(v) = 0$

$$f(v) = \min\left\{1 + \sum_{u \in \text{child}(v)} f(u)\,,\ |\text{child}(v)| + \sum_{w \in \text{grandchild}(v)} f(w)\right\}$$

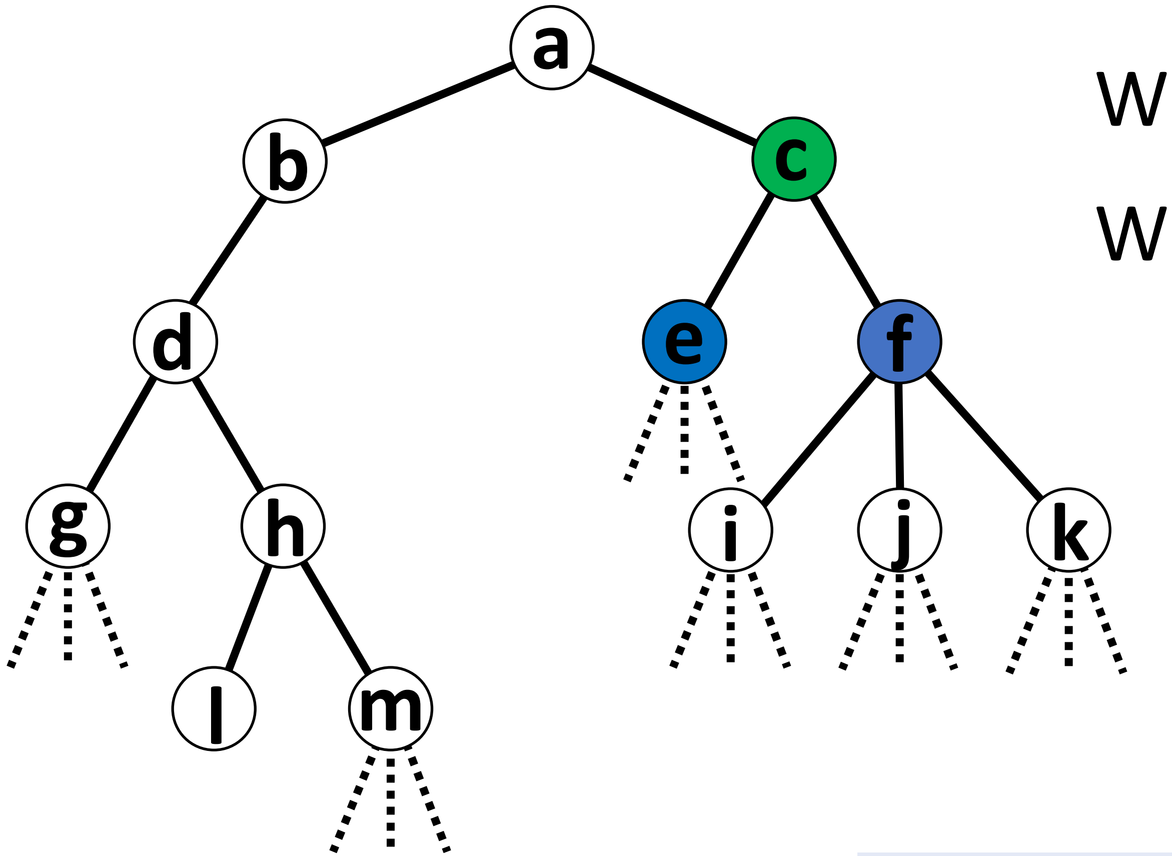
Vertex Cover in Trees



When are we ready to calculate $f(c)$?

$$f(v) = \min\left\{1 + \sum_{u \in \text{child}(v)} f(u)\,,\ |\text{child}(v)| + \sum_{w \in \text{grandchild}(v)} f(w)\right\}$$

Vertex Cover in Trees

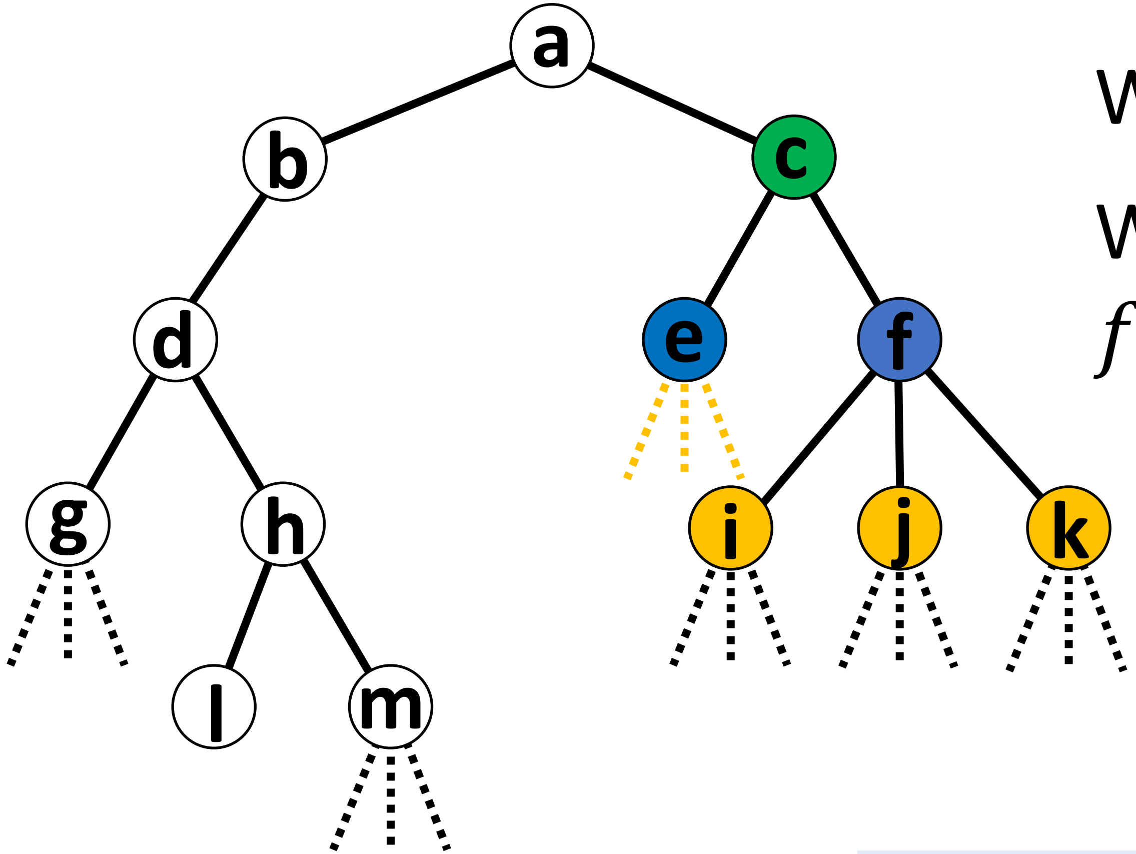


When are we ready to calculate $f(c)$?

When we have $f(e)$ and $f(f)$…

$$f(v) = \min\left\{1 + \sum_{u \in \text{child}(v)} f(u),\ |\text{child}(v)| + \sum_{w \in \text{grandchild}(v)} f(w)\right\}$$

Vertex Cover in Trees

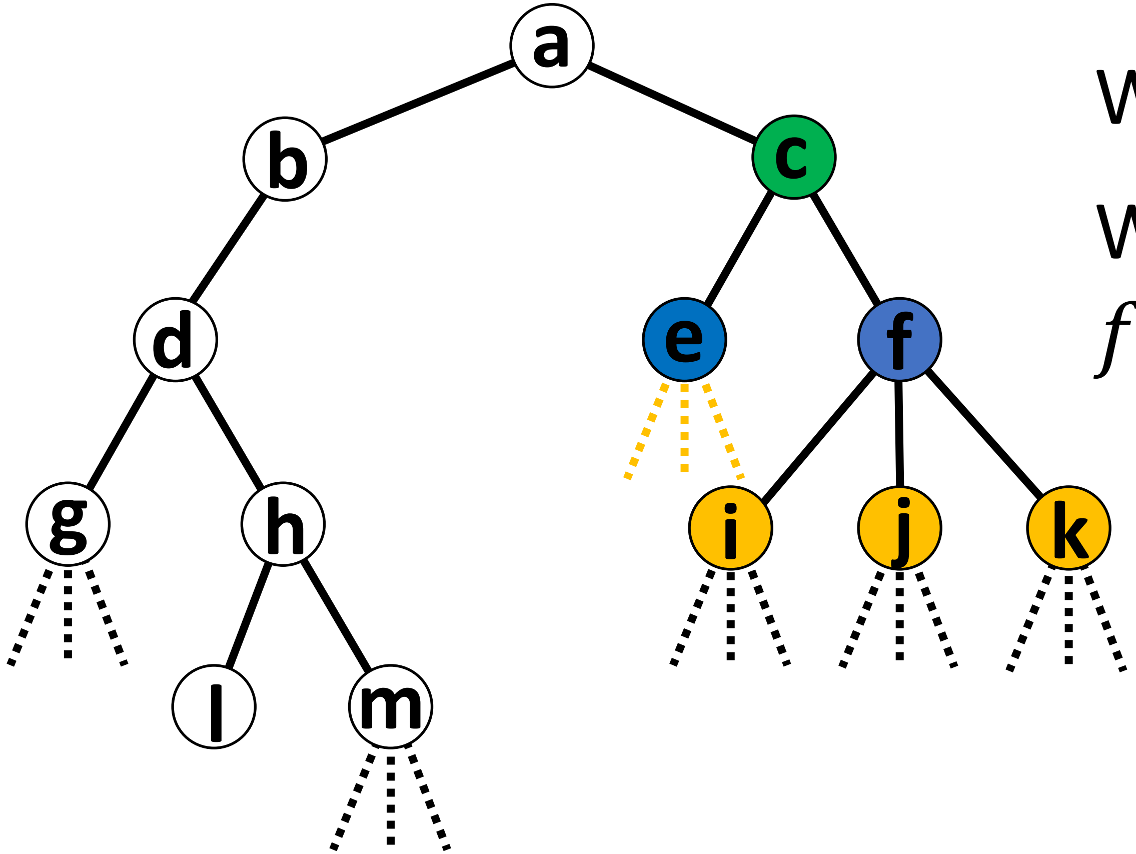


When are we ready to calculate $f(c)$?

When we have $f(e)$ and $f(f)$ and $f(v)$ for all of c's grandchildren.

$$f(v) = \min\left\{1 + \sum_{u \in \text{child}(v)} f(u),\ |\text{child}(v)| + \sum_{w \in \text{grandchild}(v)} f(w)\right\}$$

Vertex Cover in Trees



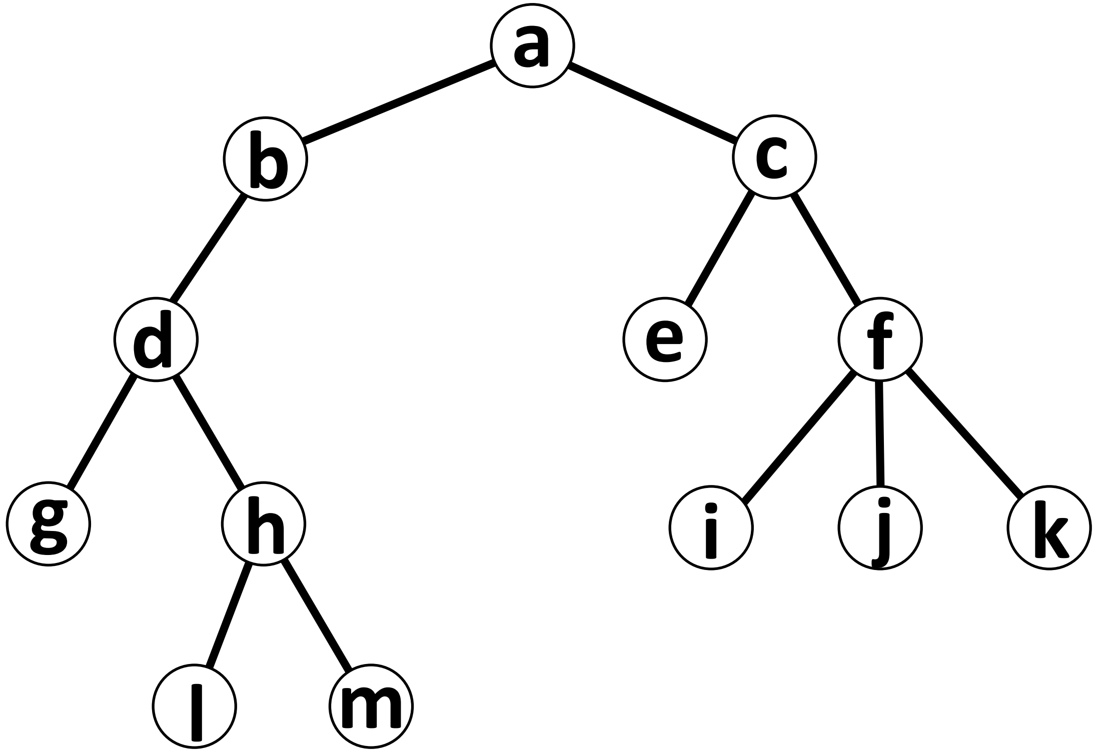
When are we ready to calculate $f(c)$?

When we have $f(e)$ and $f(f)$ and $f(v)$ for all of c's grandchildren.

How can we do this?

$$f(v) = \min\left\{1 + \sum_{u \in \text{child}(v)} f(u),\ |\text{child}(v)| + \sum_{w \in \text{grandchild}(v)} f(w)\right\}$$

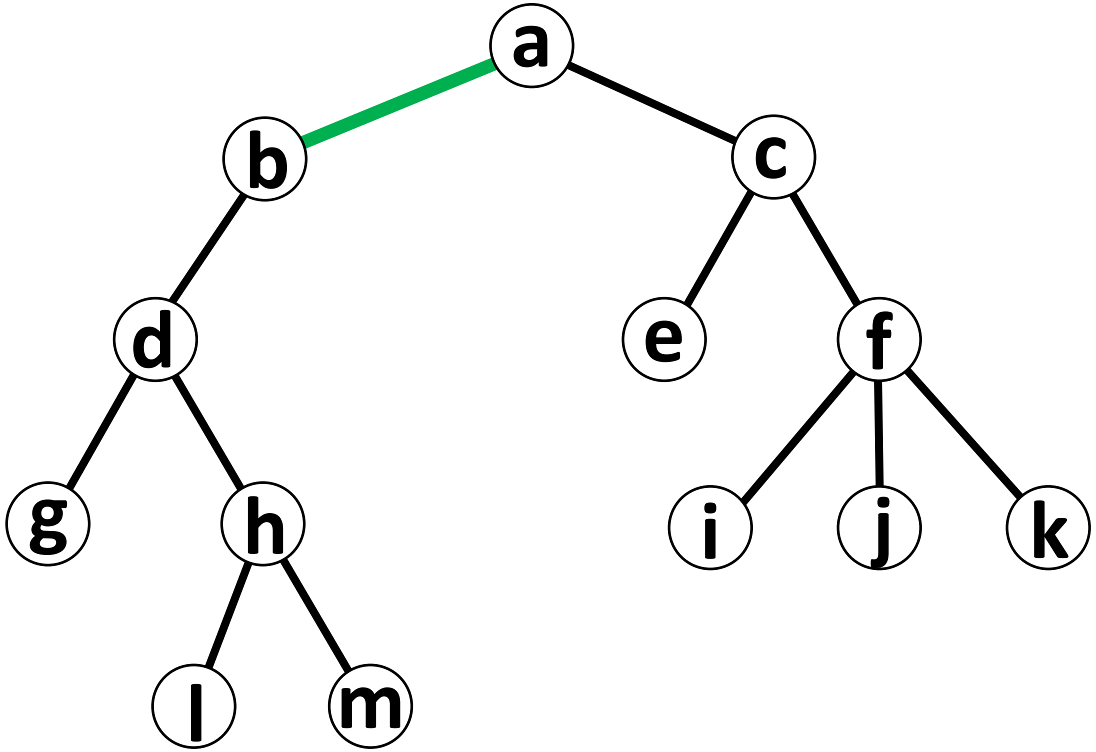
Tree Traversals



Breadth First: Start at a node. Visit all of its children, then all of its grandchildren, then great-grandchildren,…

Order visited: a, b, c, d, e, f, g, h, i, j, k, l ,m

Tree Traversals



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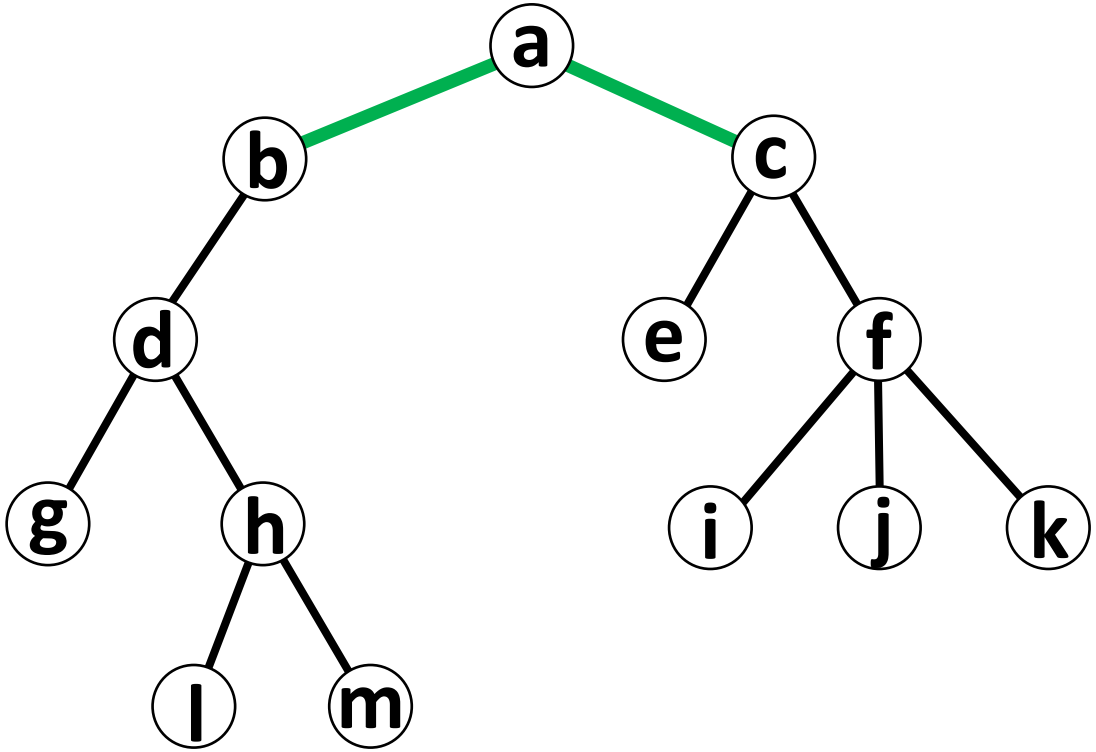

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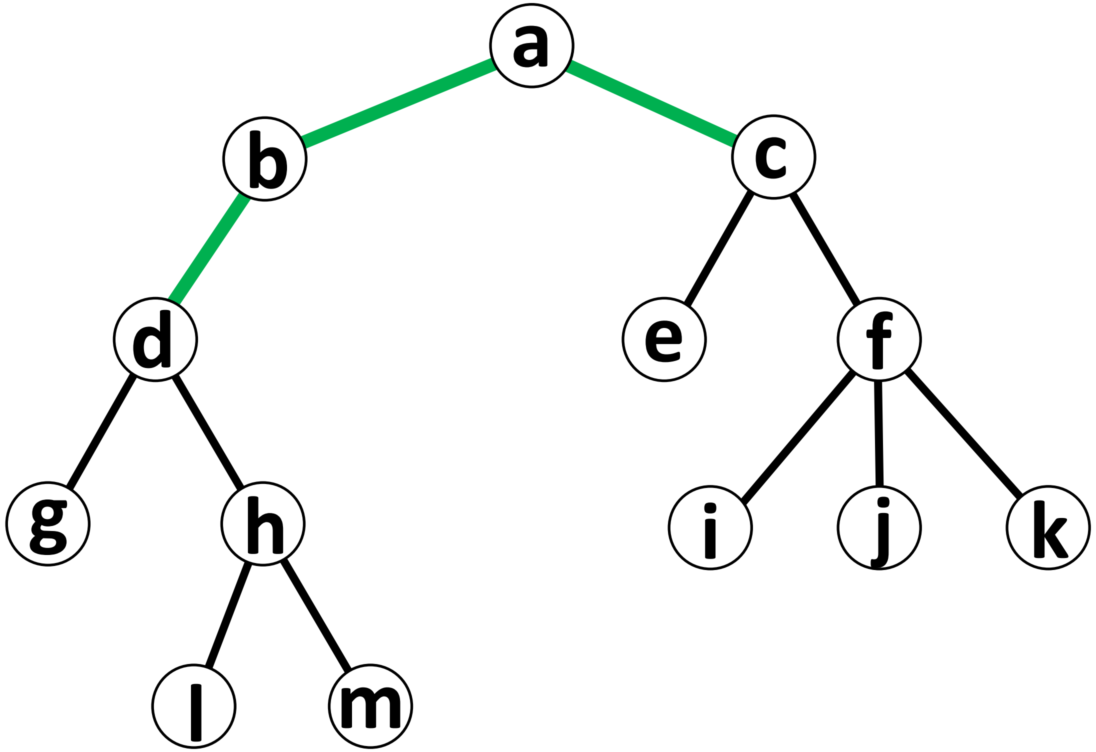

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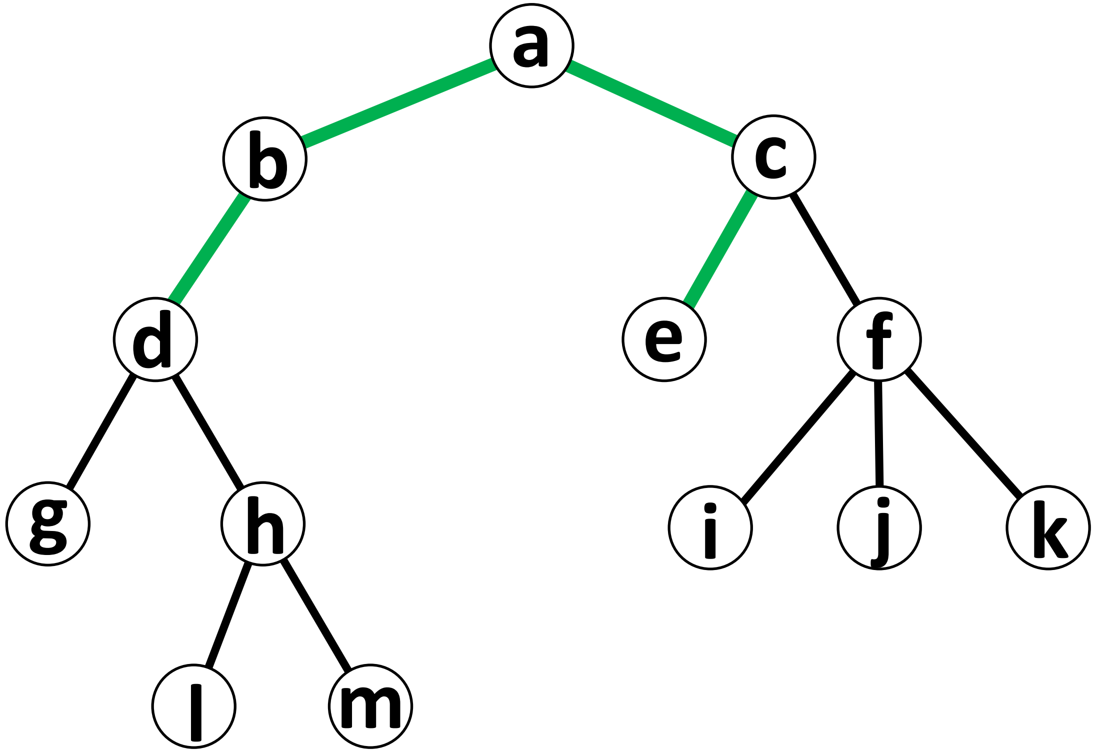

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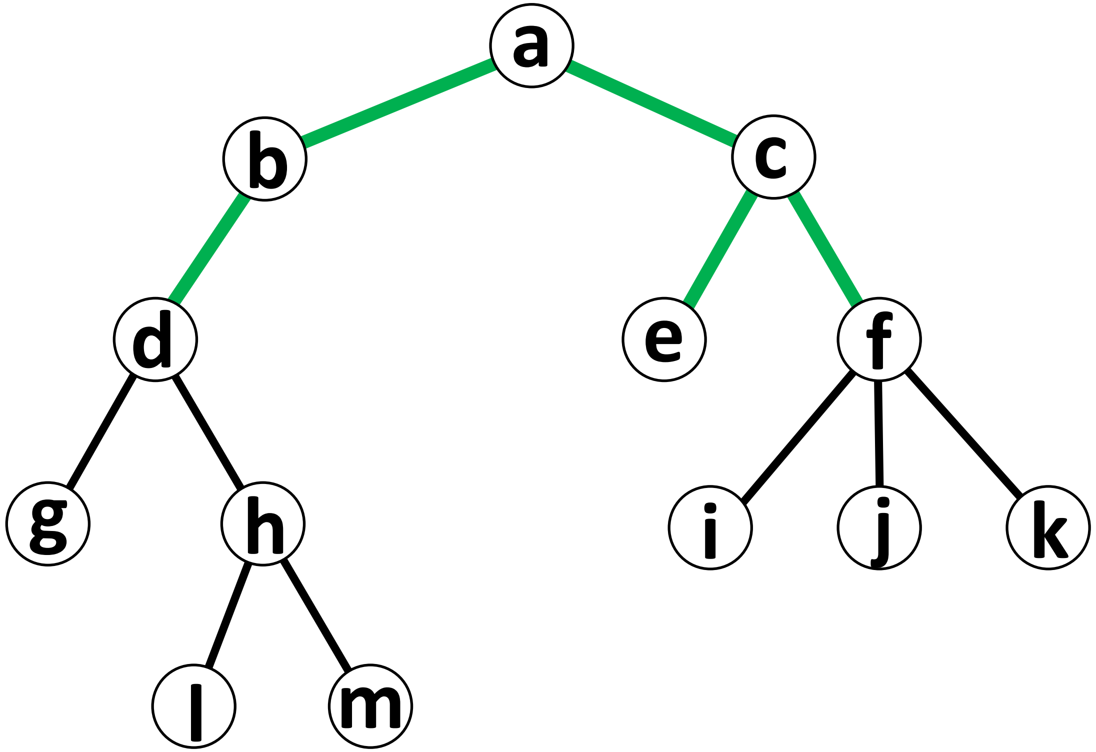

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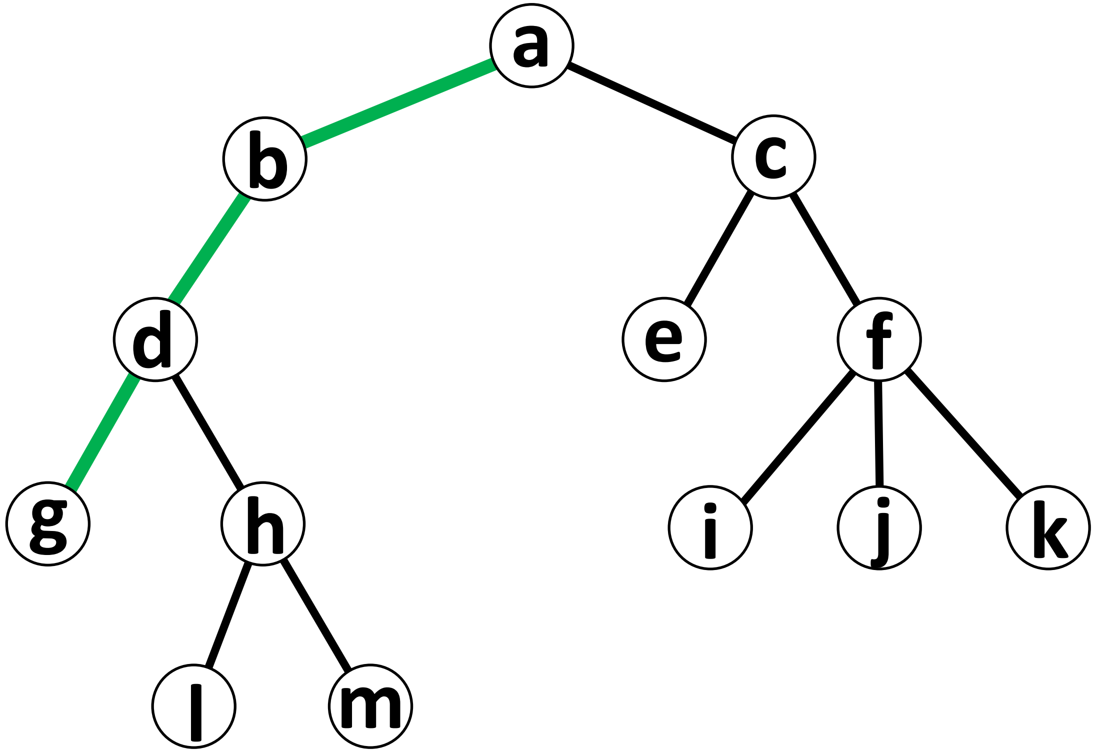
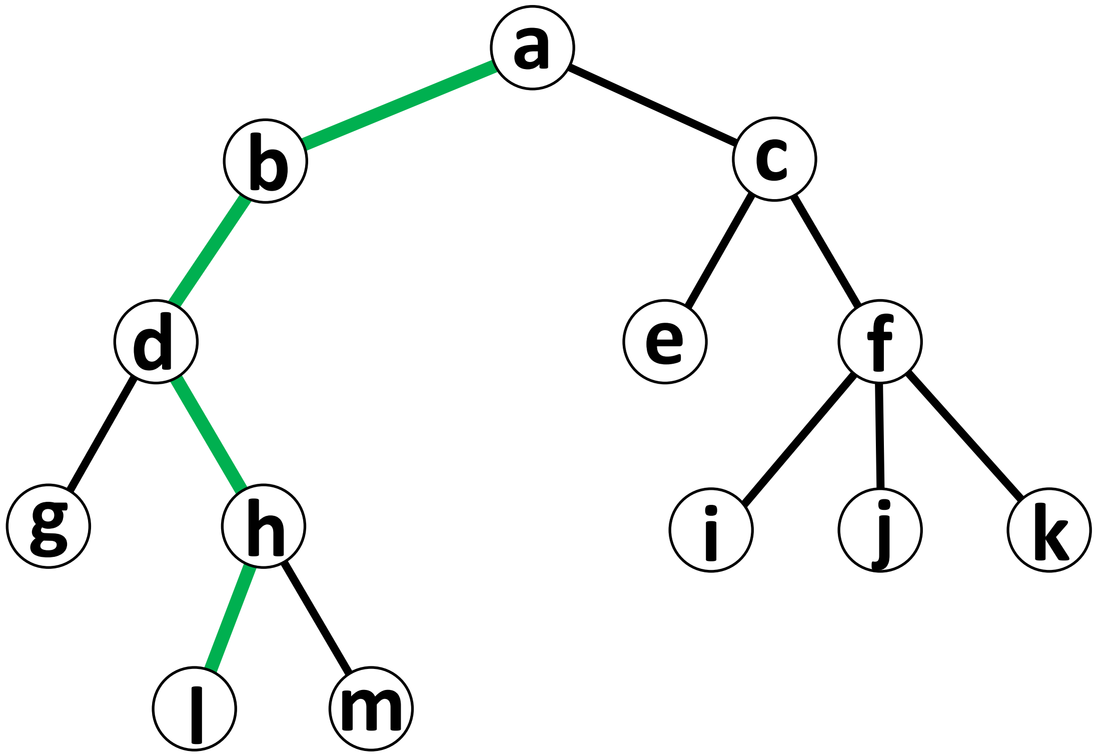
Order visited: a, b, c, d, e, f, g, h, i, j, k, l ,m

Tree Traversals



Depth First: Start at a node. Follow children until leaf is reached. Back track until unvisited child is available. Repeat…

Tree Traversals



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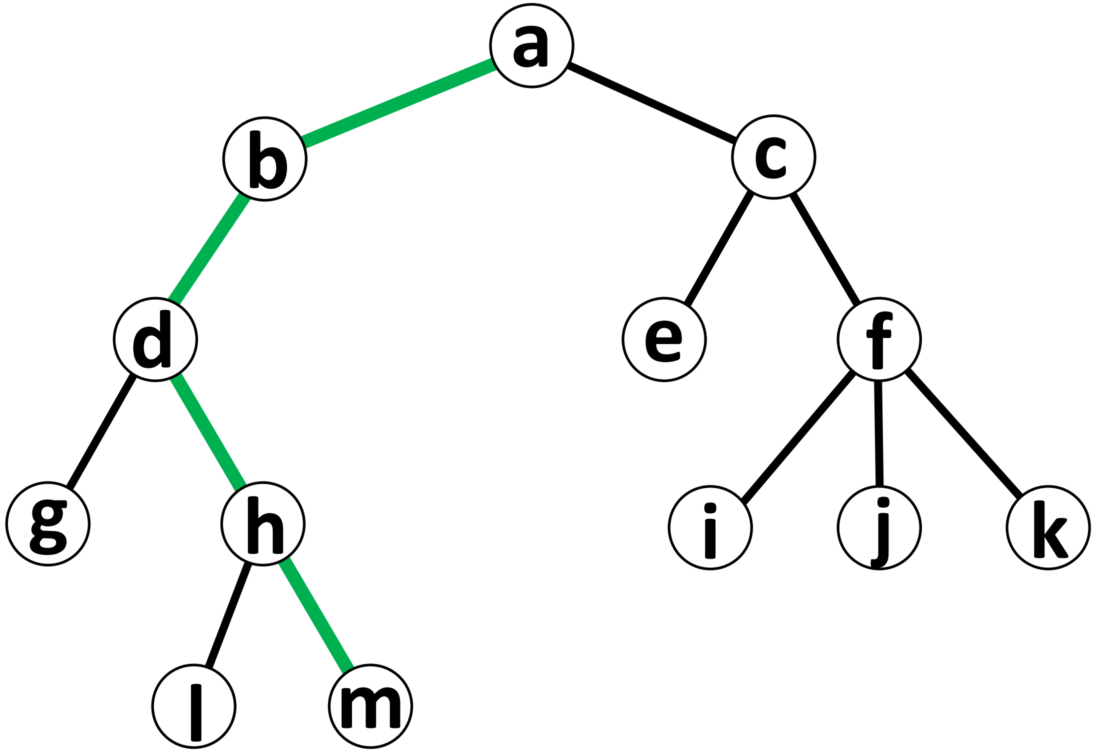
Tree Traversals



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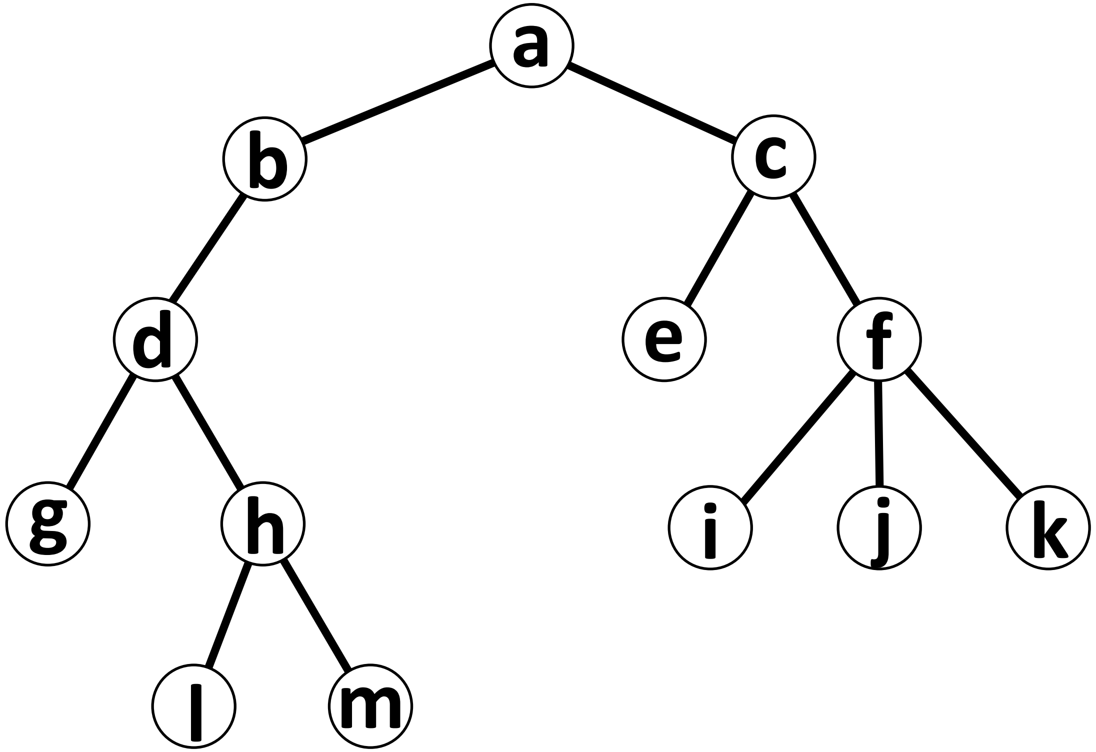
Tree Traversals

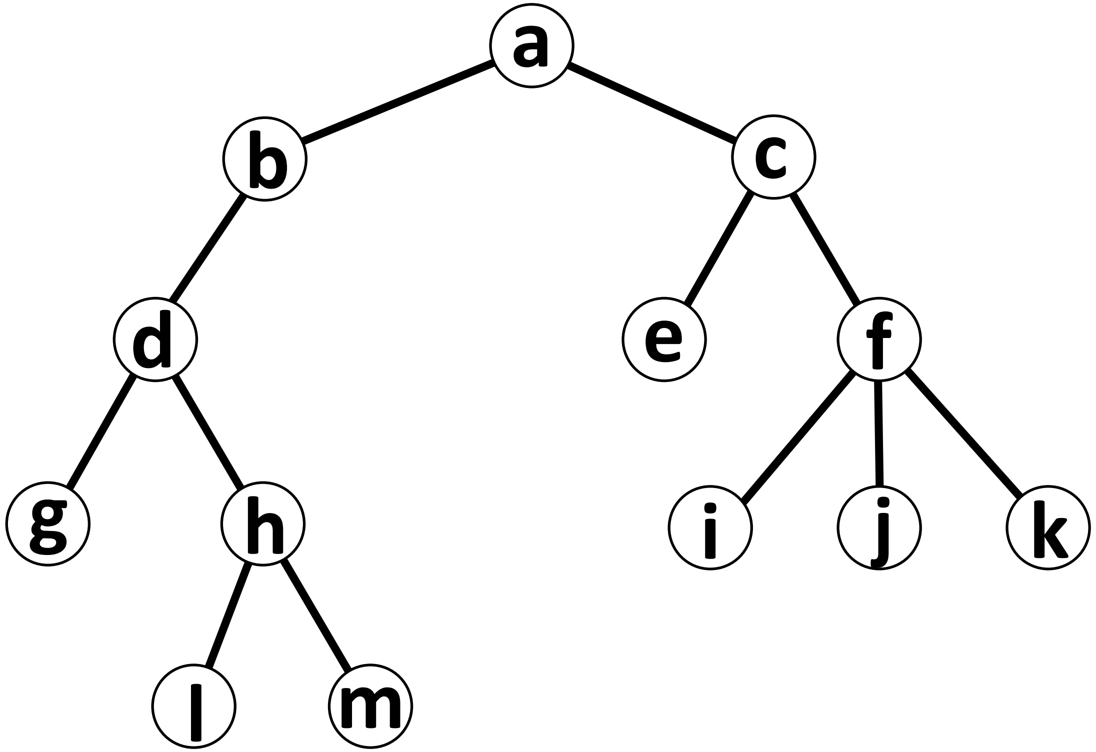


Depth First: Start at a node. Follow children until leaf is reached. Back track until unvisited child is available. Repeat…

```
preorder(Vertex v):
   if v not null:
      print(v.value())
      for u in child(v):
         preorder(u)
```

```
postorder(Vertex v):
   if v not null:
      for u in child(v):
         postorder(u)
      print(v.value())
```

Tree Traversals



Depth First: Start at a node. Follow children until leaf is reached. Back track until unvisited child is available. Repeat…

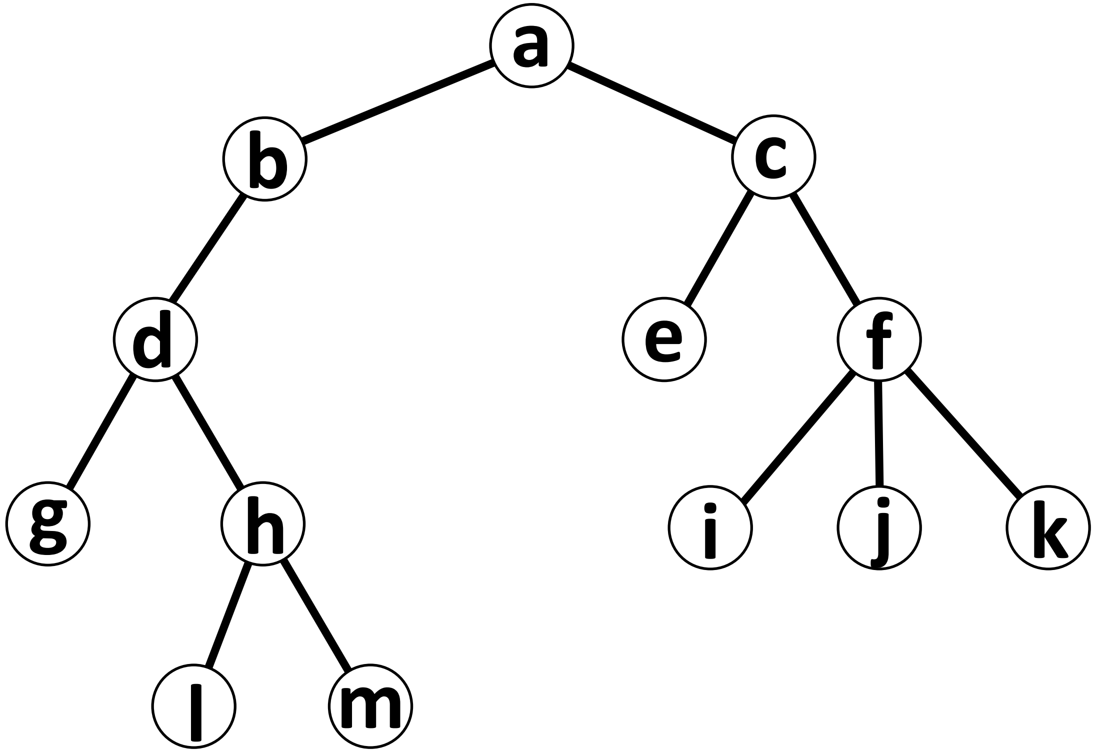
1. **Process parent**
2. **Process children**

```
preorder(Vertex v):
  if v not null:
    print(v.value())
    for u in child(v):
      preorder(u)
```

1. **Process children**
2. **Process parent**

```
postorder(Vertex v):
  if v not null:
    for u in child(v):
      postorder(u)
    print(v.value())
```

Tree Traversals



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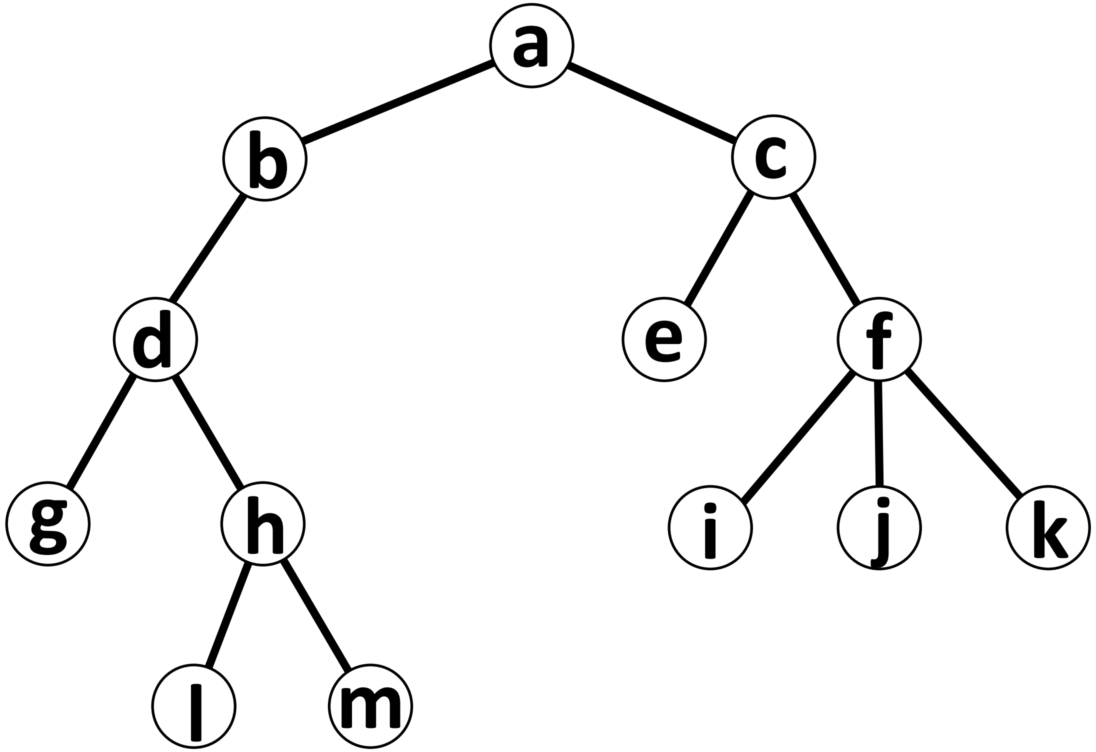
preorder(Vertex v):
    if v not null:
        print(v.value())
        for u in child(v):
            preorder(u)
```

```
postorder(Vertex v):
    if v not null:
        for u in child(v):
            postorder(u)
        print(v.value())
```

Preorder visited: a, b, d, g, h, l, m, c, e, f, i, j, k

Tree Traversals



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```
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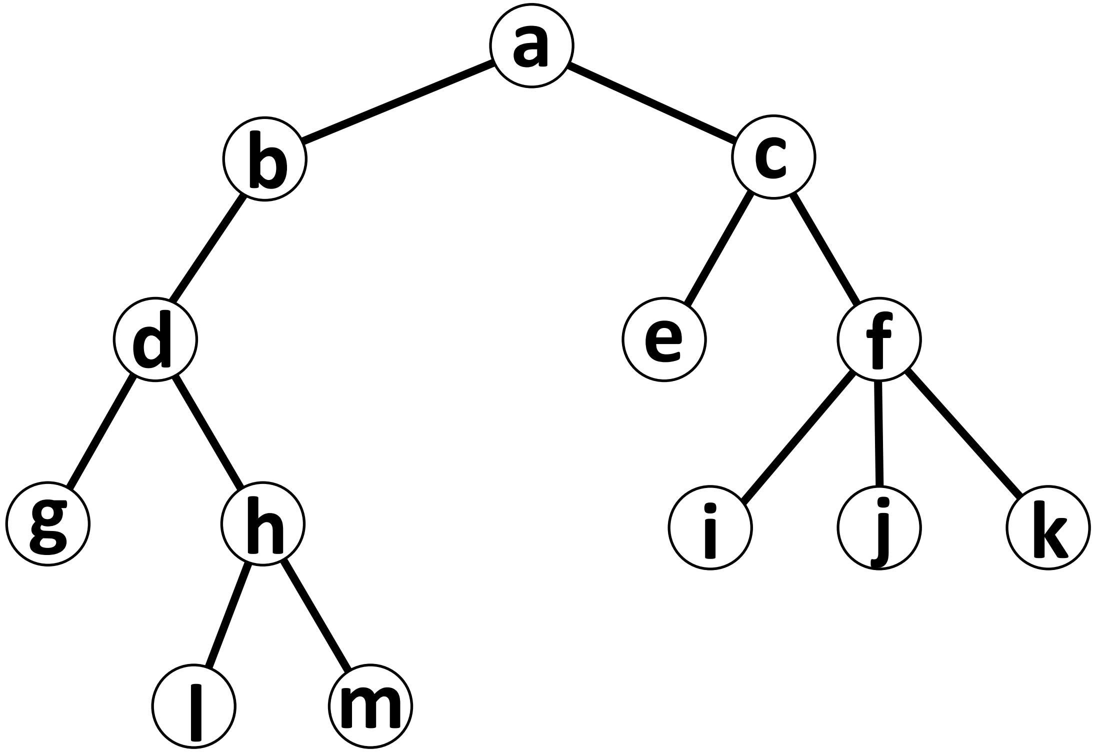
   if v not null:
      for u in child(v):
         postorder(u)
      print(v.value())
```

Preorder visited: a, b, d, g, h, l, m, c, e, f, i, j, k

Postorder visited: g, l, m, h, d, b, e, i, j, k, f, c, a

Tree Traversals

Depth First: Start at a node. Follow children until leaf is reached. Back track until unvisited child is available. Repeat…



```
preorder(Vertex v):
    if v not null:
        print(v.value())
        for u in child(v):
            preorder(u)
```

Which one do we want?

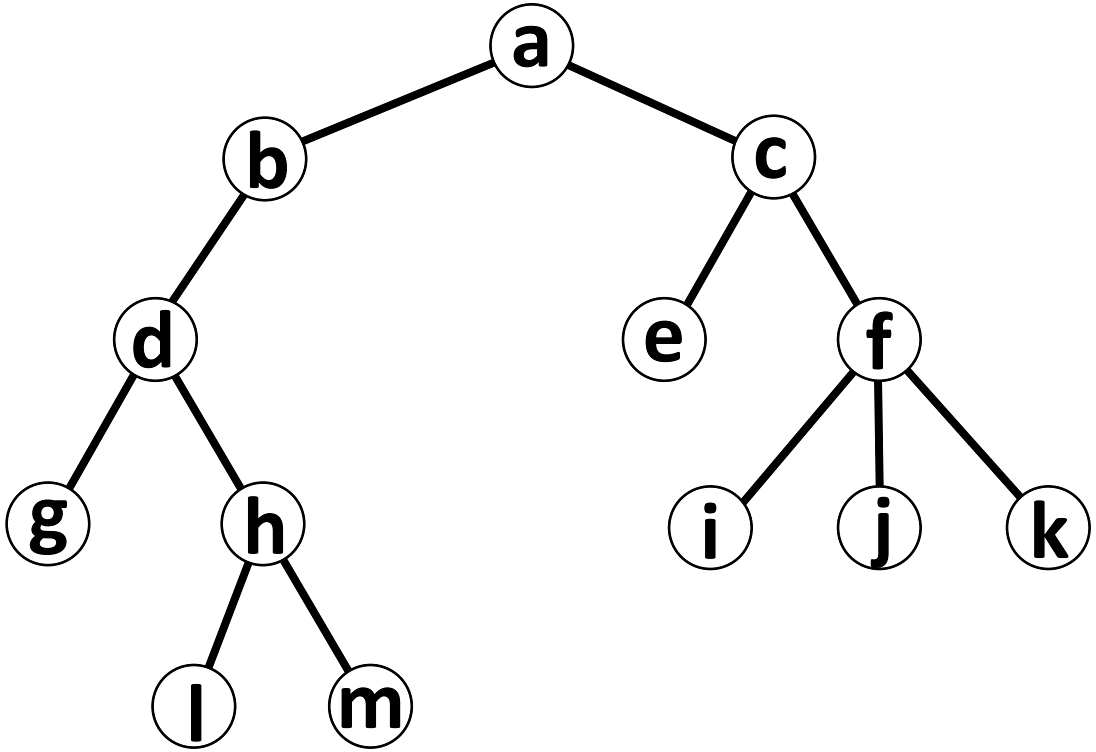
```
postorder(Vertex v):
    if v not null:
        for u in child(v):
            postorder(u)
        print(v.value())
```

Preorder visited: a, b, d, g, h, l, m, c, e, f, i, j, k

Postorder visited: g, l, m, h, d, b, e, i, j, k, f, c, a

Tree Traversals

Depth First: Start at a node. Follow children until leaf is reached. Back track until unvisited child is available. Repeat…



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postorder(vertex v):
    if v not null:
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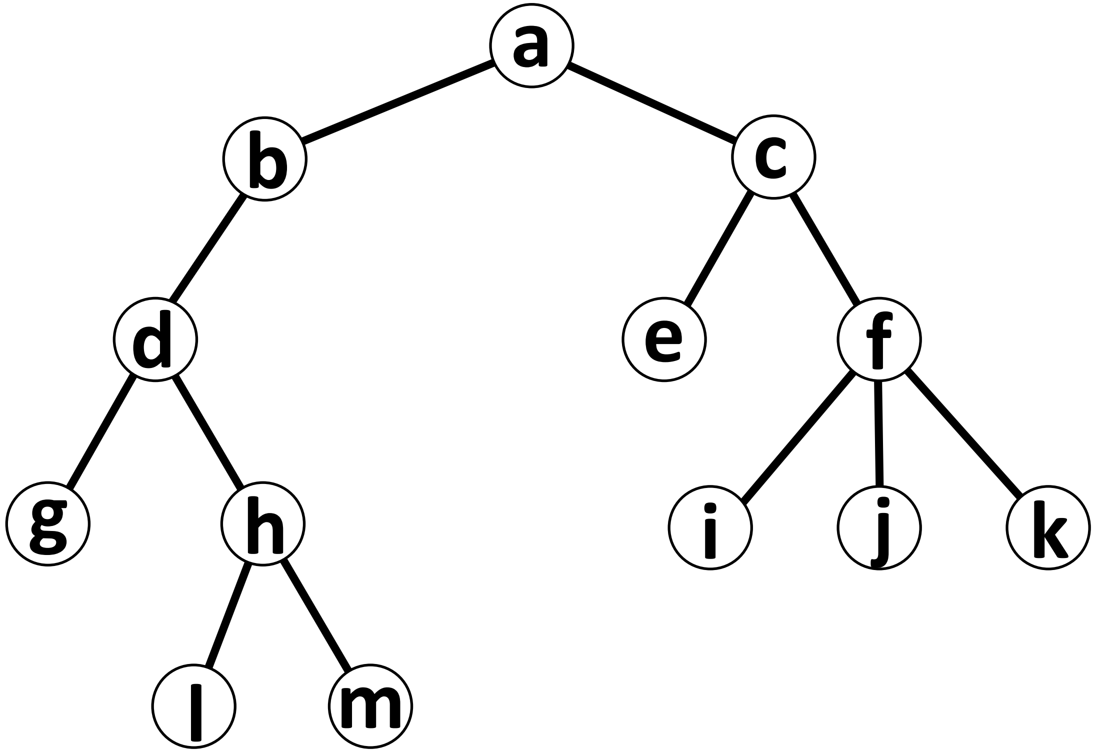
Preorder visited: a, b, d, g, h, l, m, c, e, f, i, j, k

Postorder visited: g, l, m, h, d, b, e, i, j, k, f, c, a

```
min_vc(Vertex v):
```

Vertex Cover in Trees

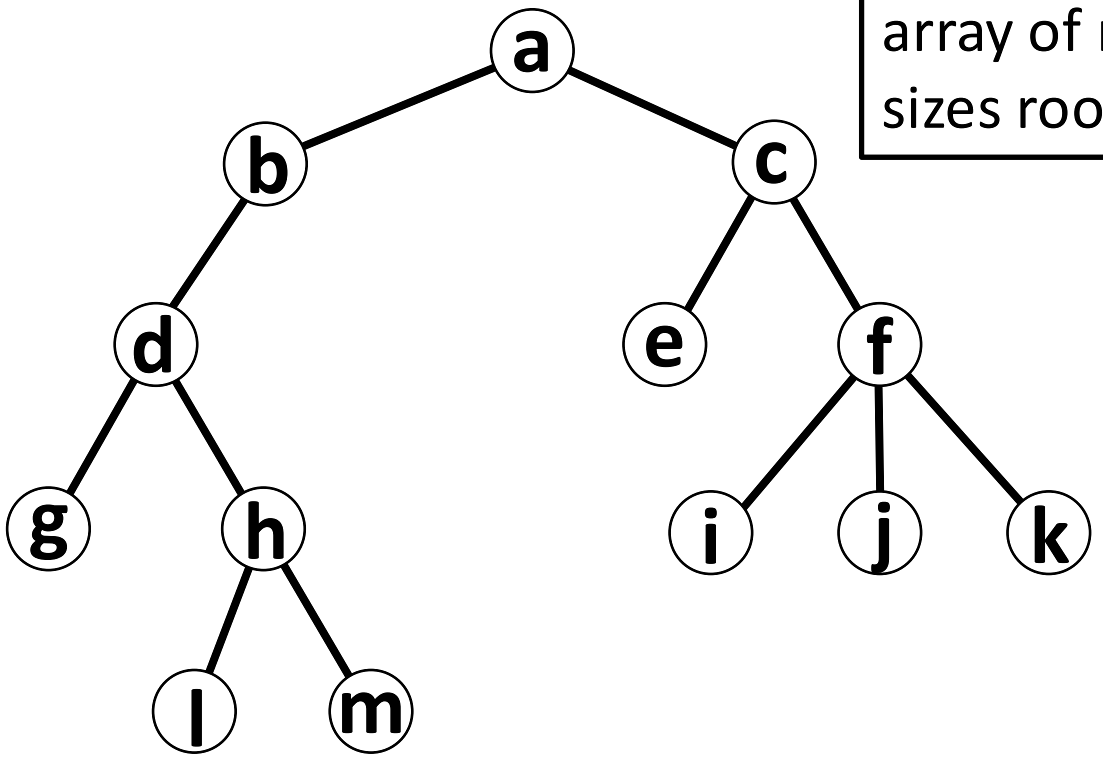


$$f(v) = \min\left\{1 + \sum_{u \in \text{child}(v)} f(u)\,,\ |\text{child}(v)| + \sum_{w \in \text{grandchild}(v)} f(w)\right\}$$

```
min_vc(Vertex v):
```

Vertex Cover in Trees

Let vc[v] be array of min VC sizes rooted at v.



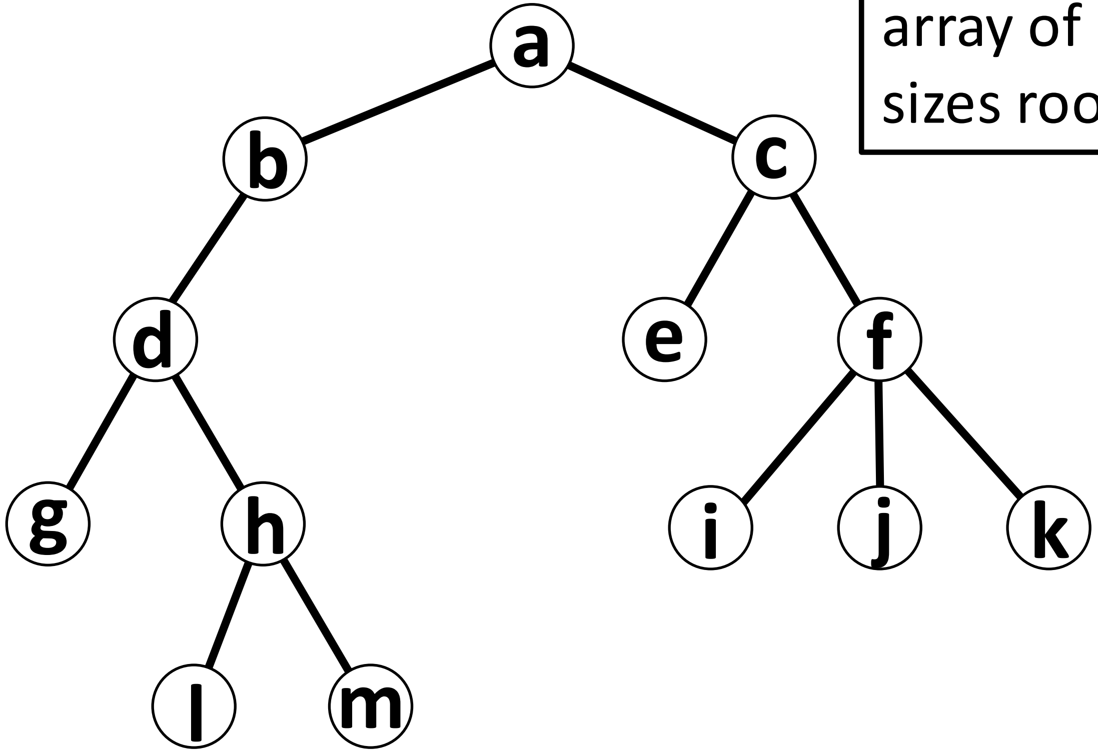
$$f(v) = \min\left\{1 + \sum_{u \in \text{child}(v)} f(u)\,,\ |\text{child}(v)| + \sum_{w \in \text{grandchild}(v)} f(w)\right\}$$

Vertex Cover in Trees

```
min_vc(Vertex v):
    if v is leaf:

    else:
```

Let vc[v] be array of min VC sizes rooted at v.



$$f(v) = \min\left\{1 + \sum_{u \in \text{child}(v)} f(u)\,,\ |\text{child}(v)| + \sum_{w \in \text{grandchild}(v)} f(w)\right\}$$

Vertex Cover in Trees

```
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   if v is leaf:
      vc[v] = 0
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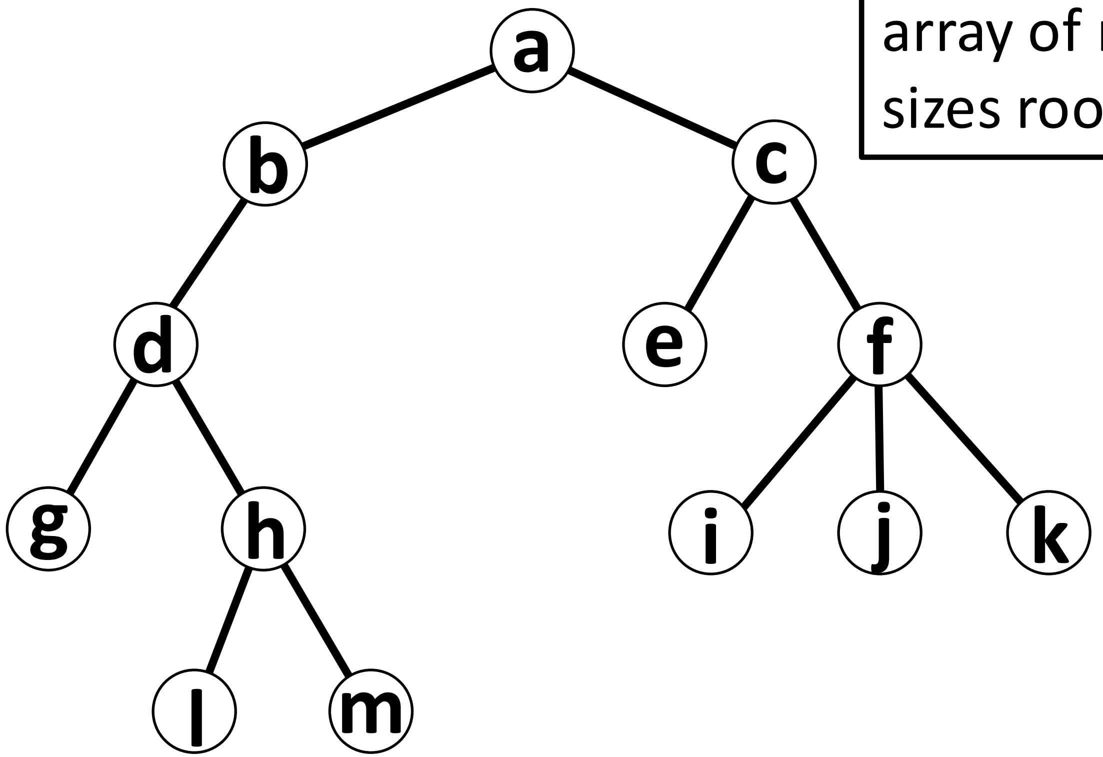

$$f(v) = \min\left\{1 + \sum_{u \in \text{child}(v)} f(u)\,,\ |\text{child}(v)| + \sum_{w \in \text{grandchild}(v)} f(w)\right\}$$

Vertex Cover in Trees

```
min_vc(Vertex v):
   if v is leaf:
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   else:
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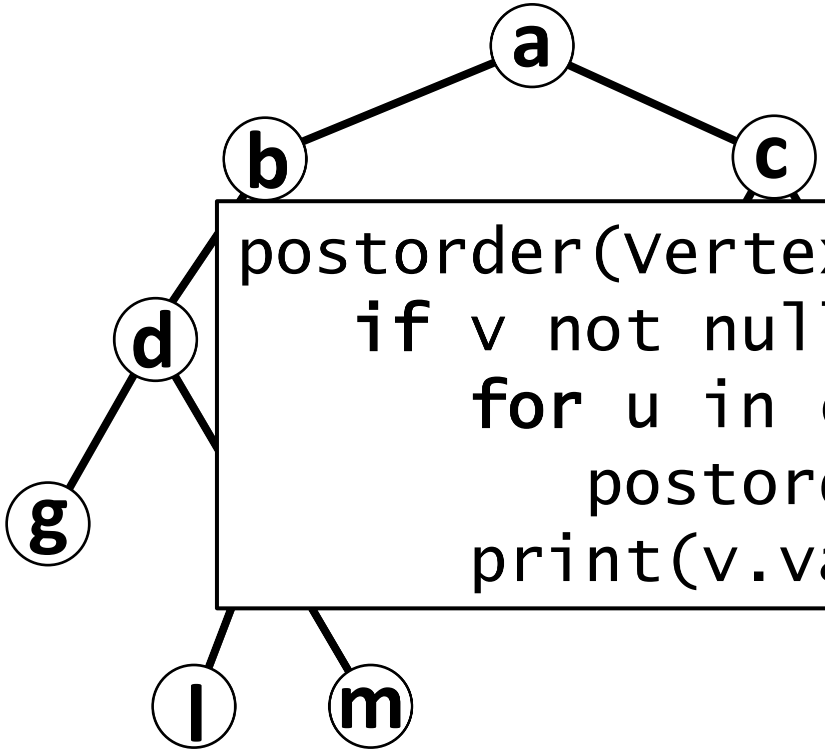
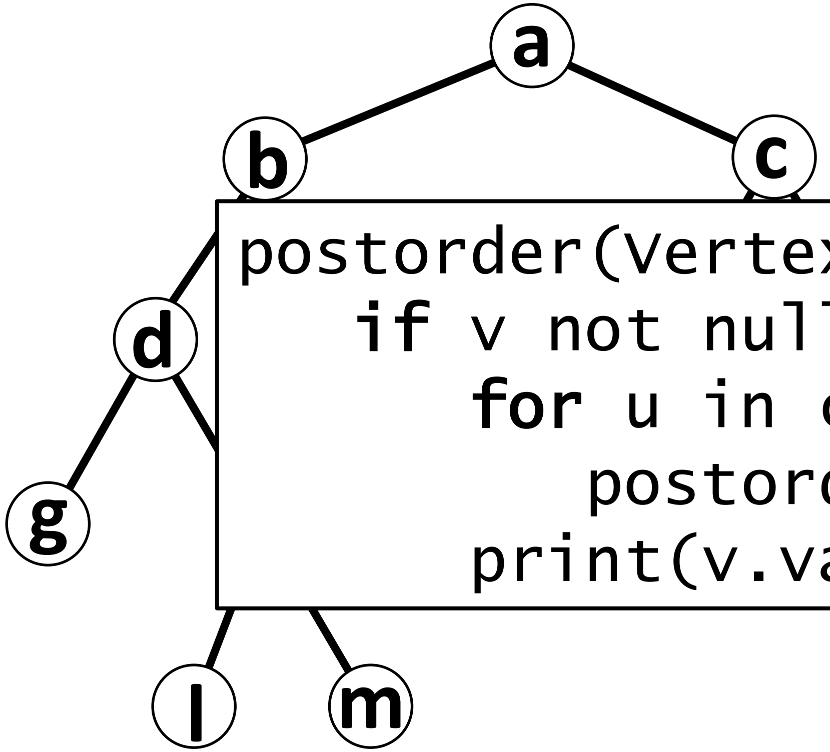
Let vc[v] be array of min VC sizes rooted at v.



```
postorder(Vertex v):
   if v not null:
      for u in child(v):
         postorder(u)
      print(v.value())
```

$$f(v) = \min\left\{1 + \sum_{u\in \text{child}(v)} f(u),\ |\text{child}(v)| + \sum_{w\in \text{grandchild}(v)} f(w)\right\}$$

Vertex Cover in Trees



```
postorder(Vertex v):
   if v not null:
      for u in child(v):
         postorder(u)
      print(v.value())
```

Let `vc[v]` be array of min VC sizes rooted at `v`.

```
min_vc(Vertex v):
   if v is leaf:
      vc[v] = 0
   else:
      // calculate descendants



      // min VC includes v



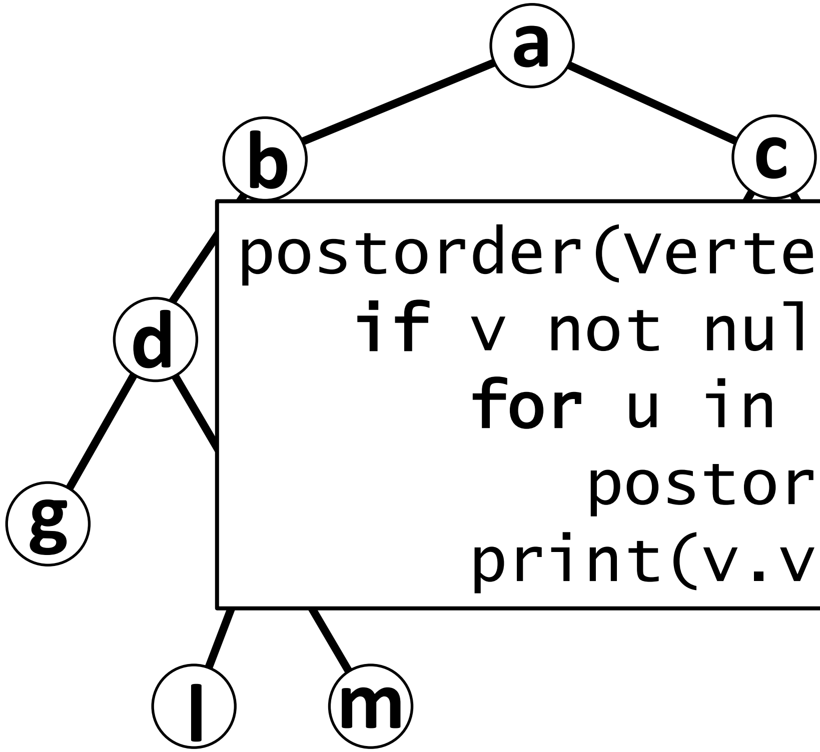
      // min VC excludes v



      vc[v] = min(???, ???)
```

$$f(v) = \min \left\{ 1 + \sum_{u \in \text{child}(v)} f(u) \, , \, |\text{child}(v)| + \sum_{w \in \text{grandchild}(v)} f(w) \right\}$$

Vertex Cover in Trees



Let vc[v] be array of min VC sizes rooted at v.

```
postorder(Vertex v):
   if v not null:
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         postorder(u)
      print(v.value())
```

```
min_vc(Vertex v):
   if v is leaf:
      vc[v] = 0
   else:
      // calculate descendants
      for u in child(v):
         min_vc(u)

      // min VC includes v



      // min VC excludes v




      vc[v] = min(???, ???)
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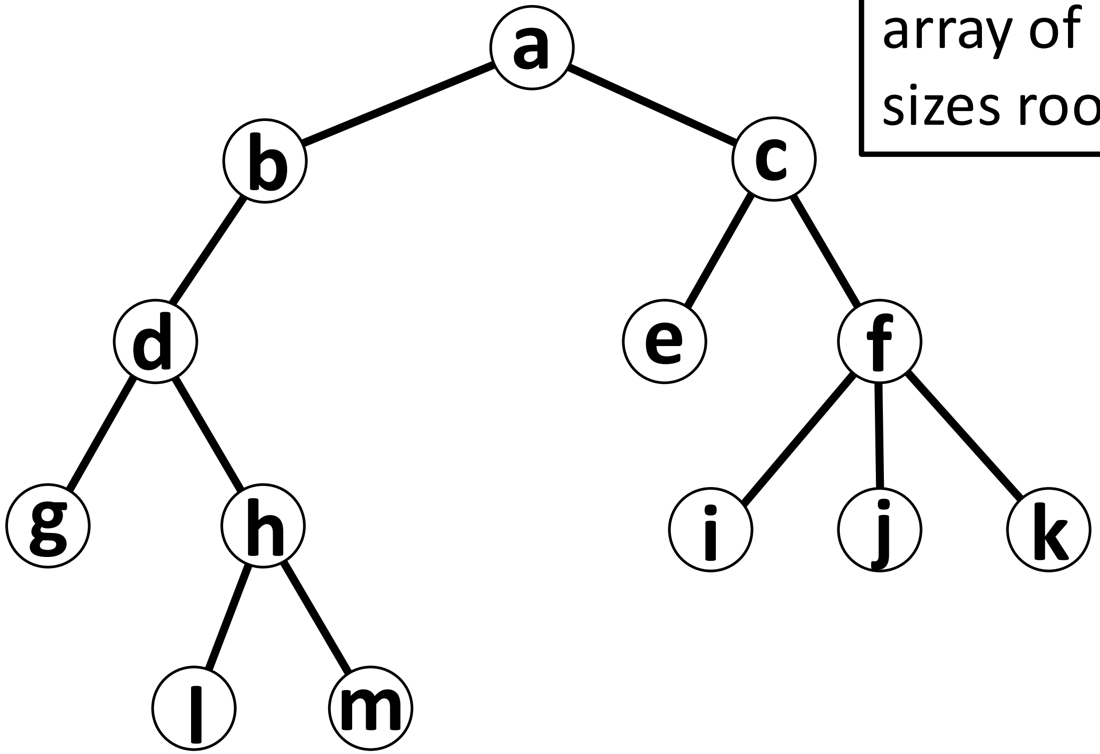
$$f(v) = \min\left\{1 + \sum_{u\in \text{child}(v)} f(u),\ |\text{child}(v)| + \sum_{w\in \text{grandchild}(v)} f(w)\right\}$$

Vertex Cover in Trees



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```
min_vc(Vertex v):
    if v is leaf:
        vc[v] = 0
    else:
        // calculate descendants
        for u in child(v):
            min_vc(u)

        // min VC includes v
        inc = 1



        // min VC excludes v

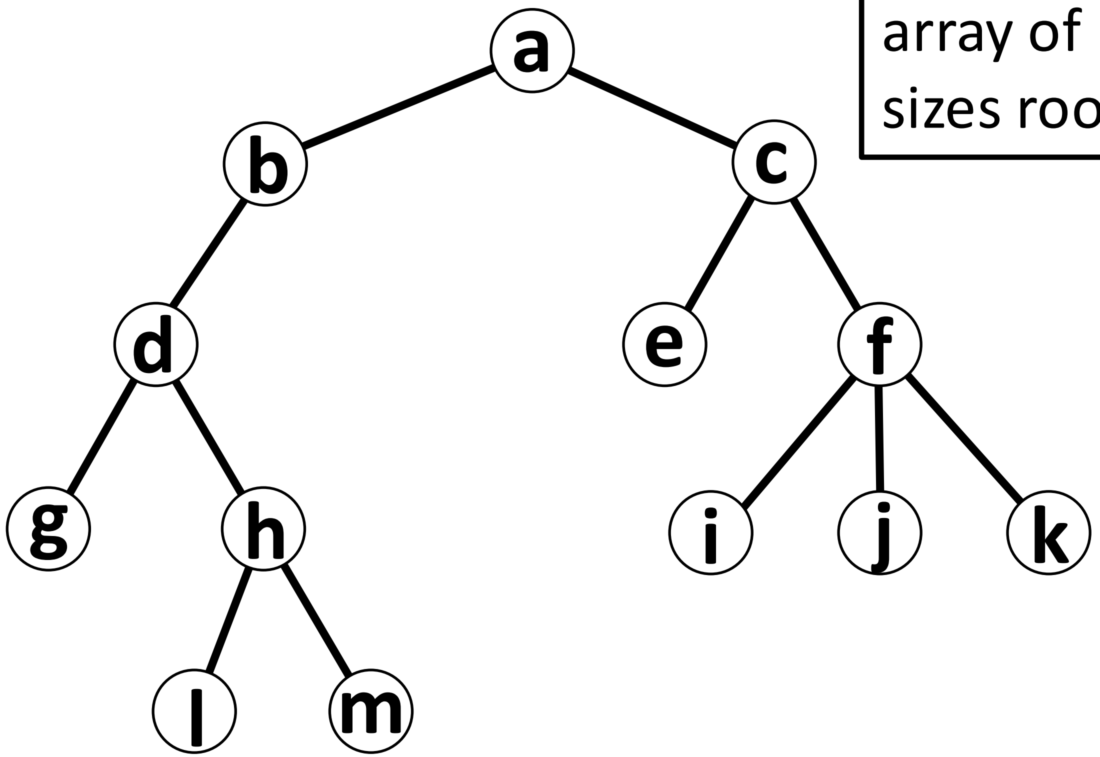




        vc[v] = min(???, ???)
```

$$f(v) = \min\left\{1 + \sum_{u \in \text{child}(v)} f(u),\ |\text{child}(v)| + \sum_{w \in \text{grandchild}(v)} f(w)\right\}$$

Vertex Cover in Trees



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      // calculate descendants
      for u in child(v):
         min_vc(u)

      // min VC includes v
      inc = 1
      for u in child(v):
         inc += vc[u]

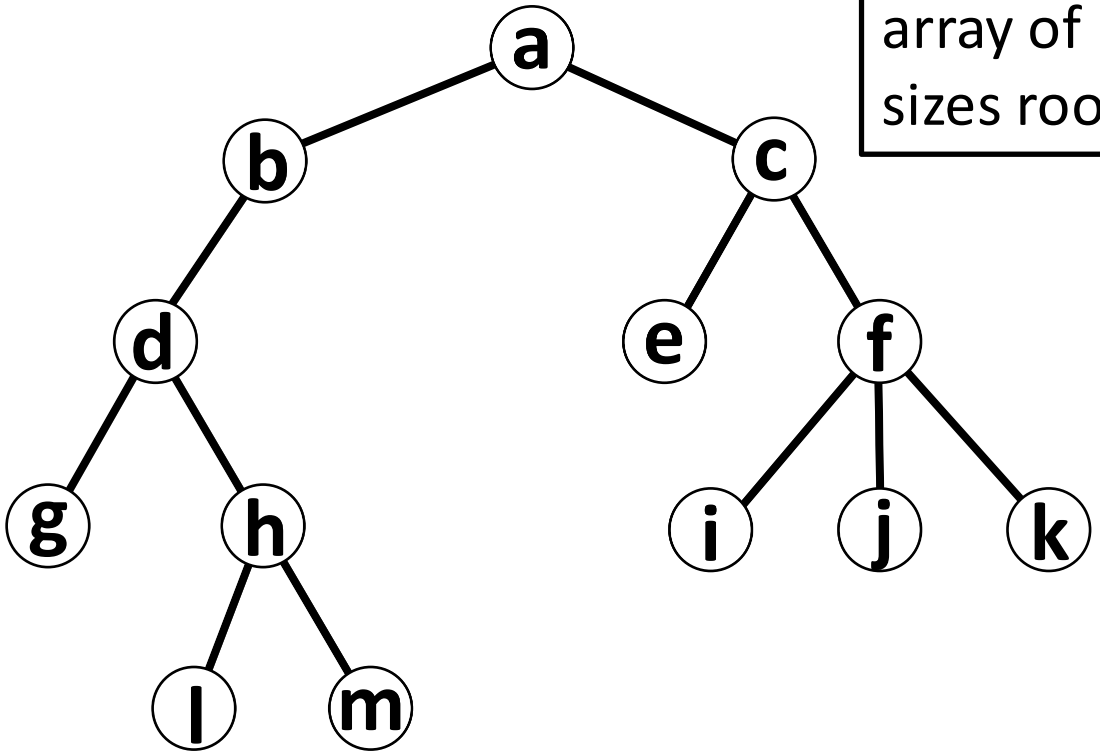
      // min VC excludes v



      vc[v] = min(???, ???)
```

$$f(v) = \min\left\{1 + \sum_{u \in \text{child}(v)} f(u),\ |\text{child}(v)| + \sum_{w \in \text{grandchild}(v)} f(w)\right\}$$

Vertex Cover in Trees



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      for u in child(v):
         min_vc(u)

      // min VC includes v
      inc = 1
      for u in child(v):
         inc += vc[u]

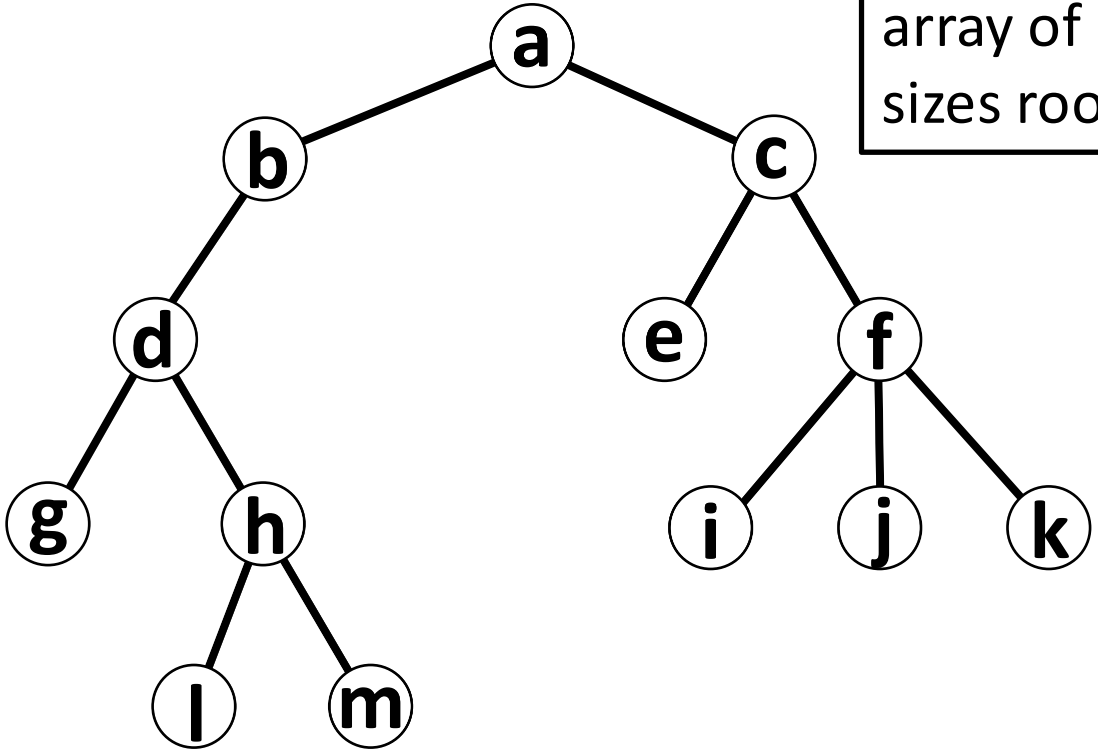
      // min VC excludes v
      exc = |child(v)|



      vc[v] = min(???, ???)
```

$$f(v) = \min\left\{1 + \sum_{u \in \text{child}(v)} f(u),\ |\text{child}(v)| + \sum_{w \in \text{grandchild}(v)} f(w)\right\}$$

Vertex Cover in Trees



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      for u in child(v):
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      inc = 1
      for u in child(v):
         inc += vc[u]

      // min VC excludes v
      exc = |child(v)|
      for u in child(v):
         for w in child(u):
            exc += vc[w]

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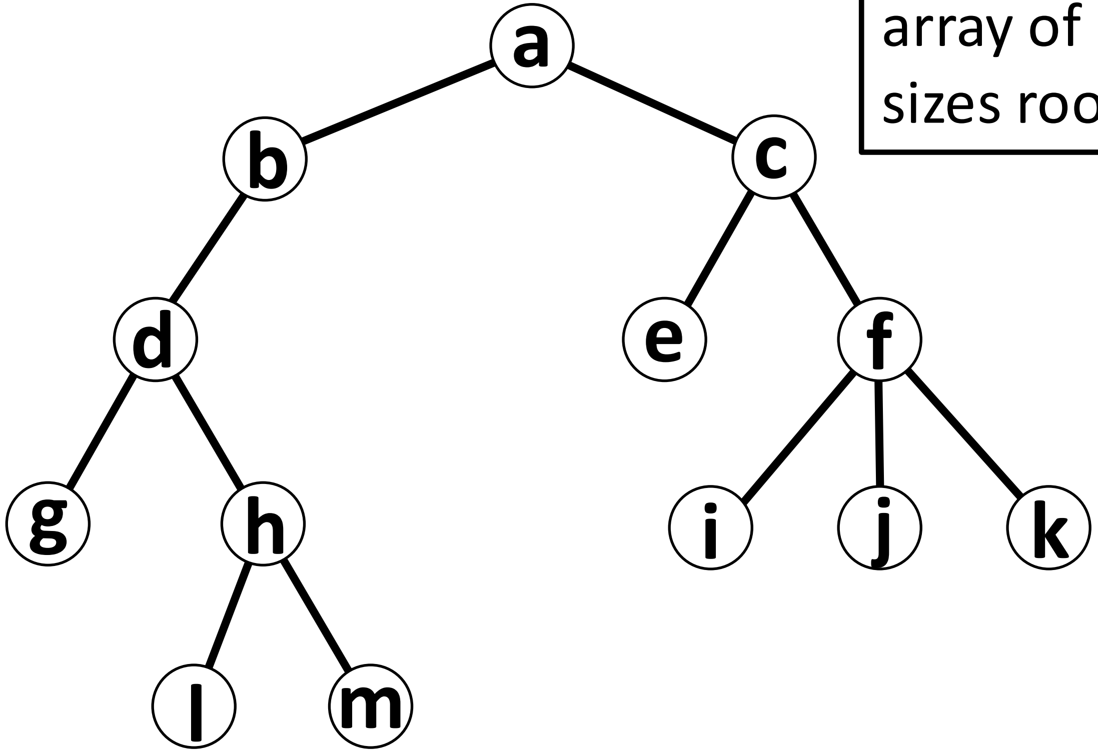
$$f(v) = \min\left\{1 + \sum_{u \in \text{child}(v)} f(u),\ |\text{child}(v)| + \sum_{w \in \text{grandchild}(v)} f(w)\right\}$$

Vertex Cover in Trees



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      // calculate descendants
      for u in child(v):
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      // min VC includes v
      inc = 1
      for u in child(v):
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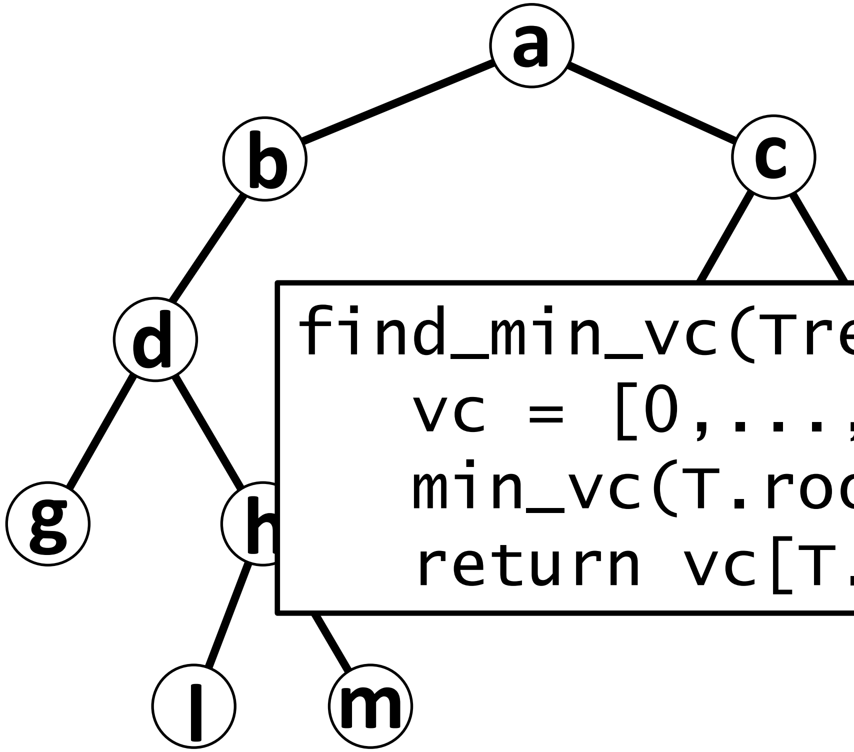
      // min VC excludes v
      exc = |child(v)|
      for u in child(v):
         for w in child(u):
            exc += vc[w]

      vc[v] = min(inc, exc)
```

$$f(v) = \min\left\{1 + \sum_{u\in\text{child}(v)} f(u),\ |\text{child}(v)| + \sum_{w\in\text{grandchild}(v)} f(w)\right\}$$

Vertex Cover in Trees

Let vc[v] be array of min VC sizes rooted at v.



```
find_min_vc(Tree T):
    vc = [0,...,0]
    min_vc(T.root)
    return vc[T.root]
```

```
min_vc(Vertex v):
    if v is leaf:
        vc[v] = 0
    else:
        // calculate descendants
        for u in child(v):
            min_vc(u)

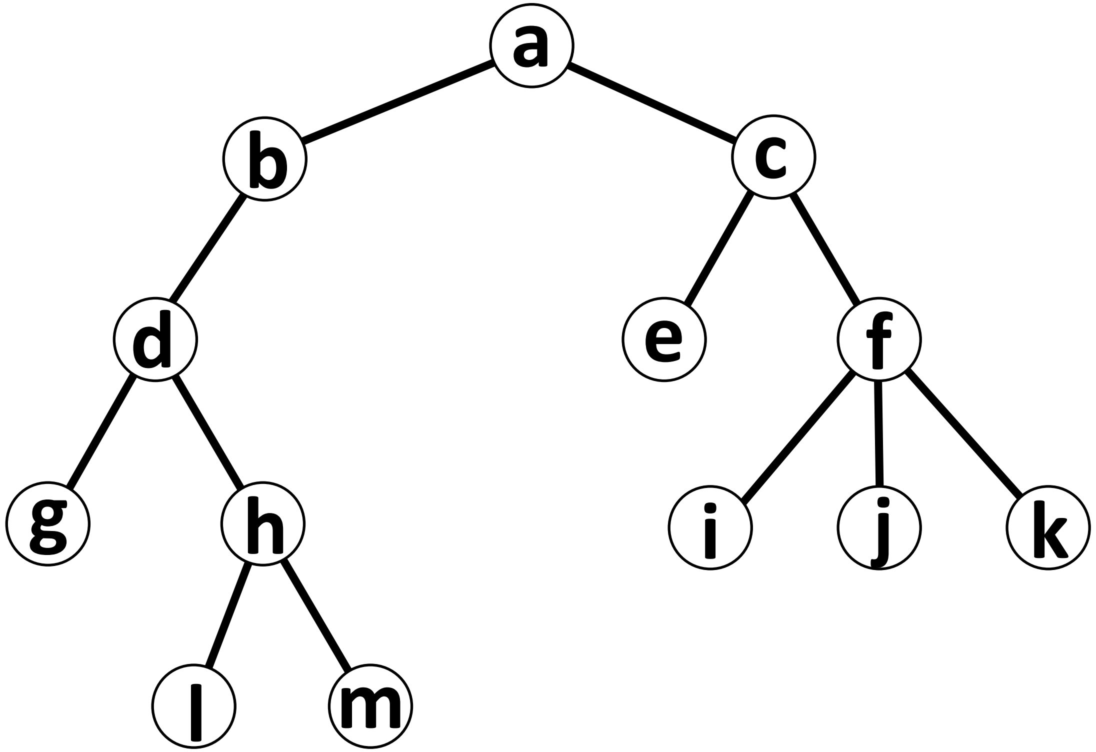
        // min VC includes v
        inc = 1
        for u in child(v):
            inc += vc[u]

        // min VC excludes v
        exc = |child(v)|
        for u in child(v):
            for w in child(u):
                exc += vc[w]

        vc[v] = min(inc, exc)
```

$$f(v) = \min\left\{1 + \sum_{u \in \text{child}(v)} f(u),\ |\text{child}(v)| + \sum_{w \in \text{grandchild}(v)} f(w)\right\}$$

Vertex Cover in Trees



Running Time: ??

```
min_vc(Vertex v):
   if v is leaf:
      vc[v] = 0
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      // calculate descendants
      for u in child(v):
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      // min VC includes v
      inc = 1
      for u in child(v):
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         for w in child(u):
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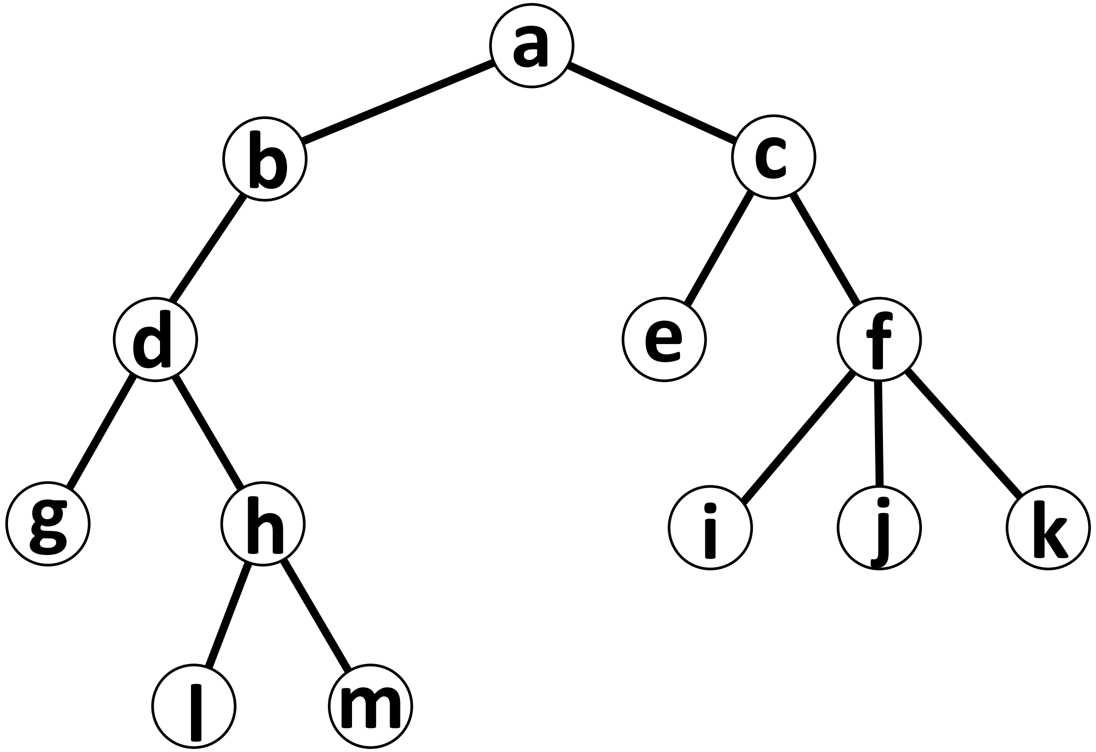
```

Vertex Cover in Trees



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        // calculate descendants
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        // min VC includes v
        inc = 1
        for u in child(v):
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        exc = |child(v)|
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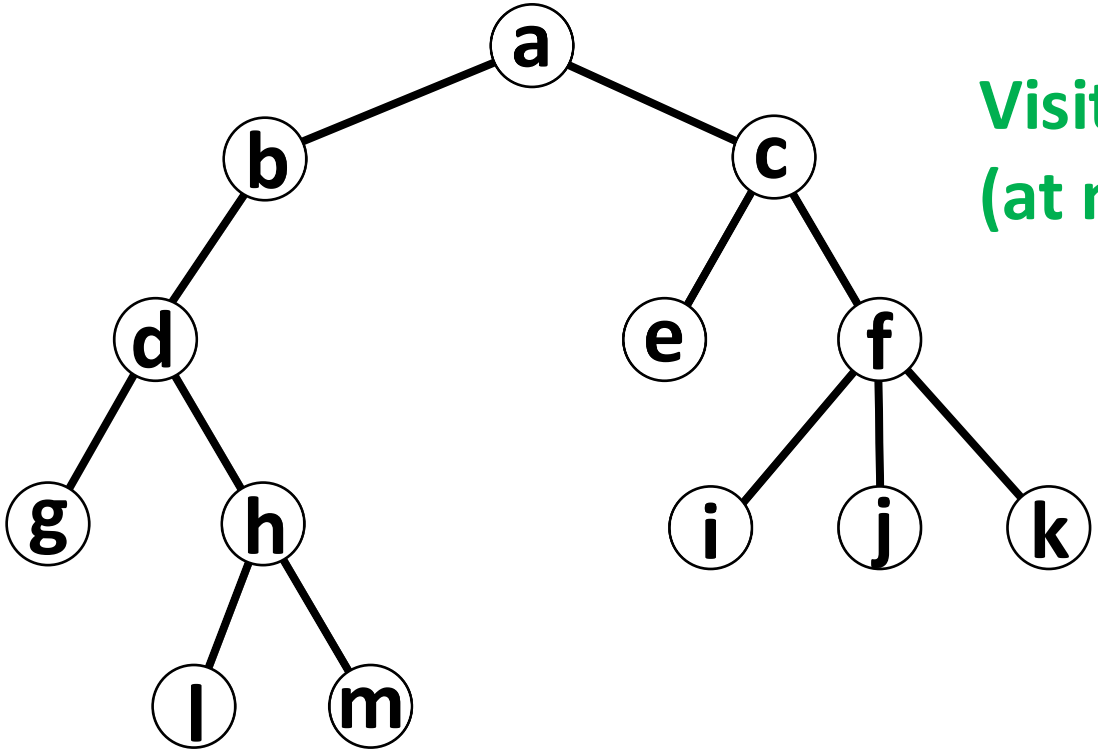
        vc[v] = min(inc, exc)
```

Running Time: ??

Vertex Cover in Trees



Running Time: ??

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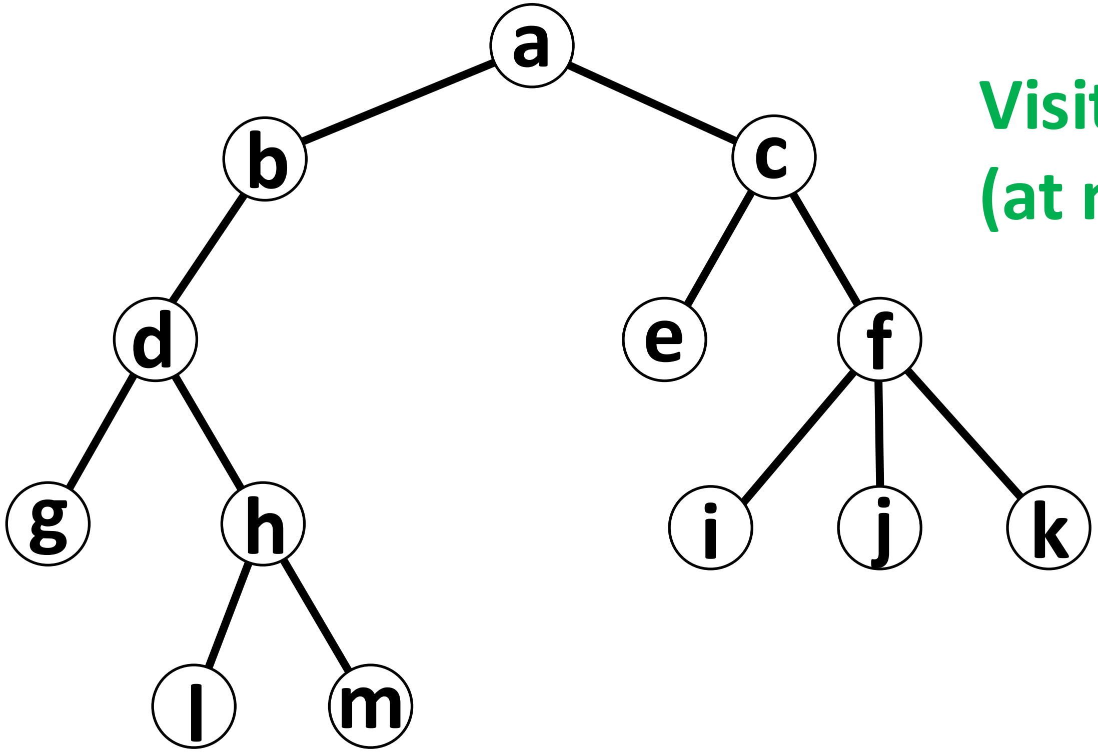
        for u in child(v):
            for w in child(u):
                exc += vc[w]

        vc[v] = min(inc, exc)
```

Visit each node (at most) once

Vertex Cover in Trees



Running Time: ??

```
min_vc(Vertex v):
    if v is leaf:
        vc[v] = 0
    else:
        // calculate descendants
        for u in child(v):
            min_vc(u)
```

Visit each node (at most) once

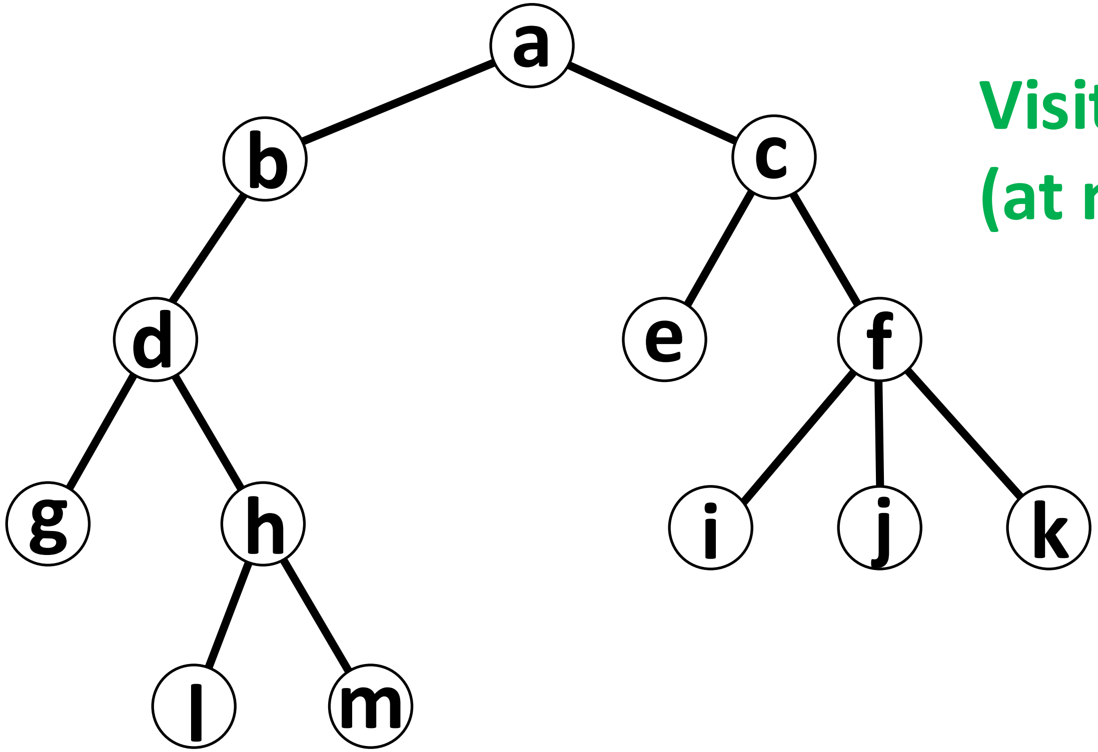
```
        // min VC includes v
        inc = 1
        for u in child(v):
            inc += vc[u]
```

??

```
        // min VC excludes v
        exc = |child(v)|
        for u in child(v):
            for w in child(u):
                exc += vc[w]

        vc[v] = min(inc, exc)
```

Vertex Cover in Trees



Running Time: ??

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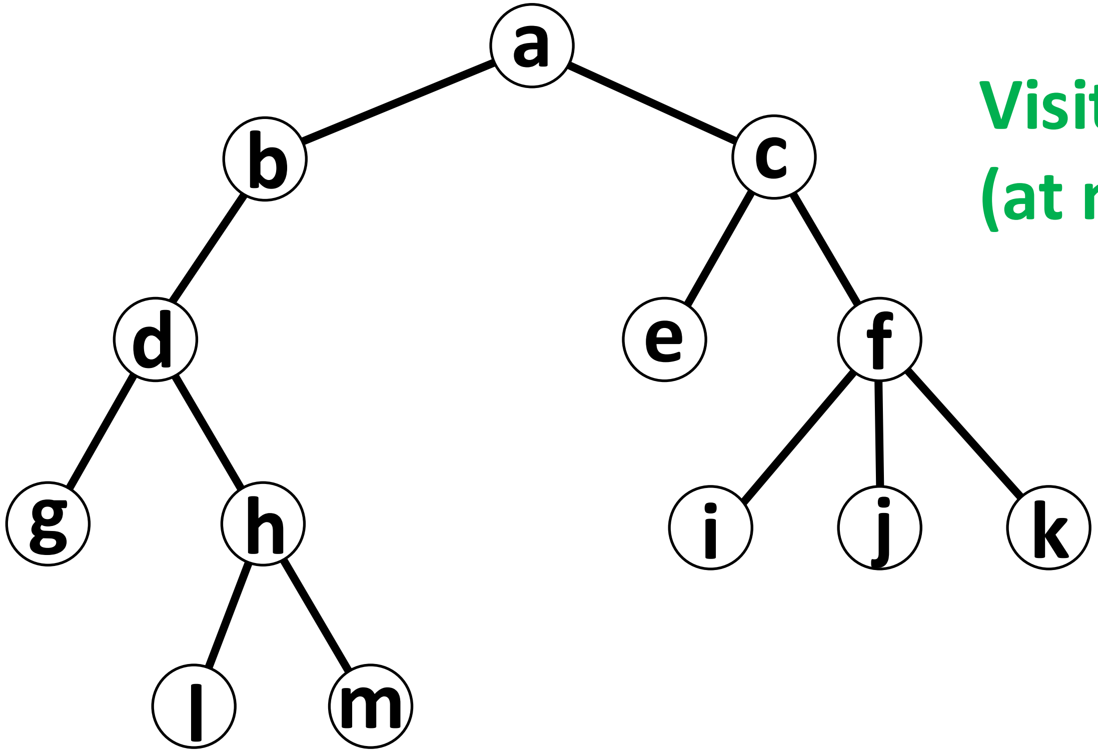
Visit each node (at most) once

??

Vertex Cover in Trees



Running Time: $O(n)$

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min_vc(Vertex v):
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Visit each node (at most) once

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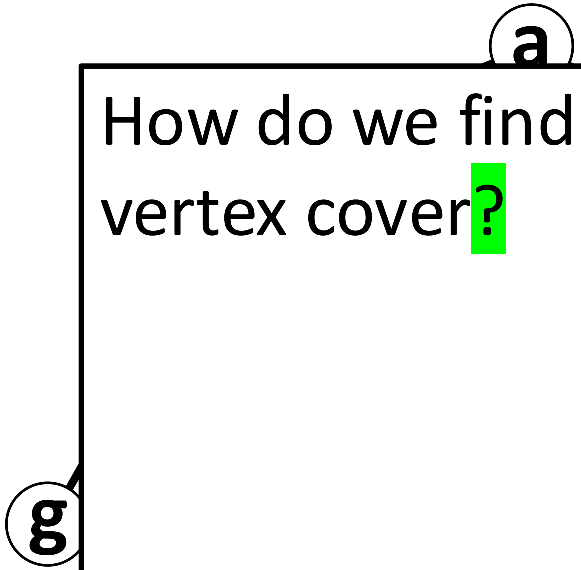
Vertex Cover in Trees



How do we find the actual minimum vertex cover?

Running Time: $O(n)$

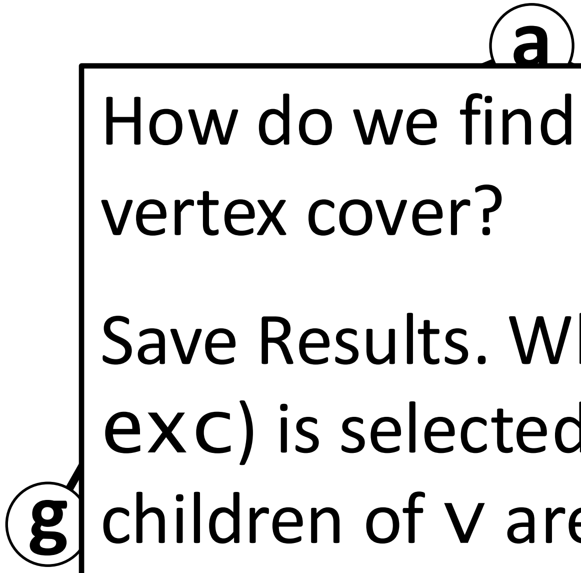
```
min_vc(Vertex v):
   if v is leaf:
      vc[v] = 0
   else:
      // calculate descendants
      for u in child(v):
         min_vc(u)

      // min VC includes v
      inc = 1
      for u in child(v):
         inc += vc[u]

      // min VC excludes v
      exc = |child(v)|
      for u in child(v):
         for w in child(u):
            exc += vc[w]

      vc[v] = min(inc, exc)
```

Vertex Cover in Trees



How do we find the actual minimum vertex cover?

Save Results. When the minimum of (`inc`, `exc`) is selected, save whether `v` or children of `v` are part of the cover.

Running Time: $O(n)$

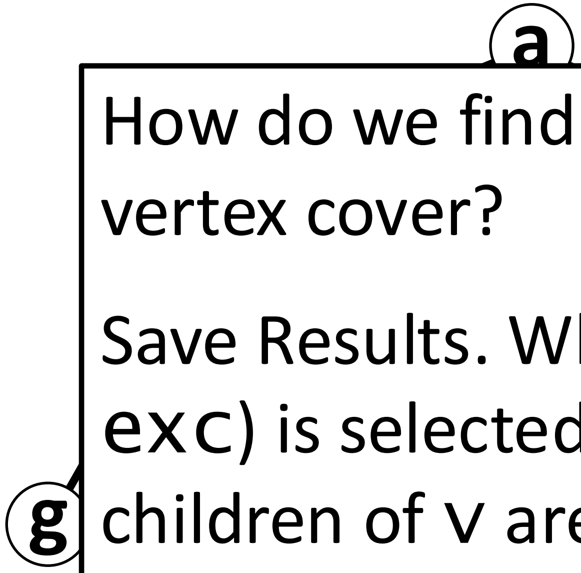
```
min_vc(Vertex v):
   if v is leaf:
      vc[v] = 0
   else:
      // calculate descendants
      for u in child(v):
         min_vc(u)

      // min VC includes v
      inc = 1
      for u in child(v):
         inc += vc[u]

      // min VC excludes v
      exc = |child(v)|
      for u in child(v):
         for w in child(u):
            exc += vc[w]

      vc[v] = min(inc, exc)
```

Vertex Cover in Trees



How do we find the actual minimum vertex cover?

Save Results. When the minimum of (`inc`, `exc`) is selected, save whether `v` or children of `v` are part of the cover.

Backtrack. At root, determine if we did `inc` or `exc` then check children (`inc`) or grandchildren (`exc`).

Running Time: $O(n)$

```
min_vc(Vertex v):
   if v is leaf:
      vc[v] = 0
   else:
      // calculate descendants
      for u in child(v):
         min_vc(u)

      // min VC includes v
      inc = 1
      for u in child(v):
         inc += vc[u]

      // min VC excludes v
      exc = |child(v)|
      for u in child(v):
         for w in child(u):
            exc += vc[w]

      vc[v] = min(inc, exc)
```