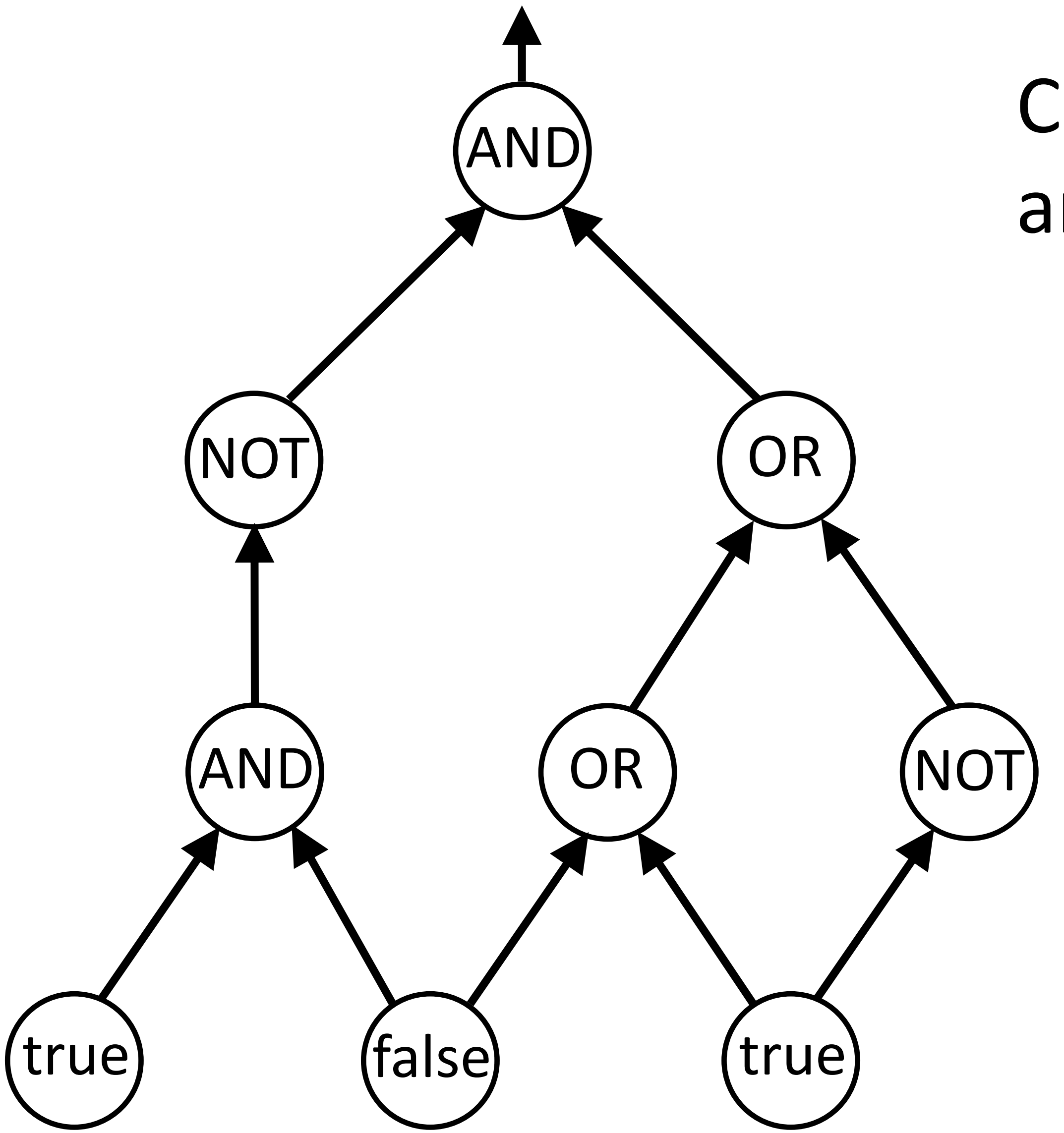
Linear Programming
CSCI 532

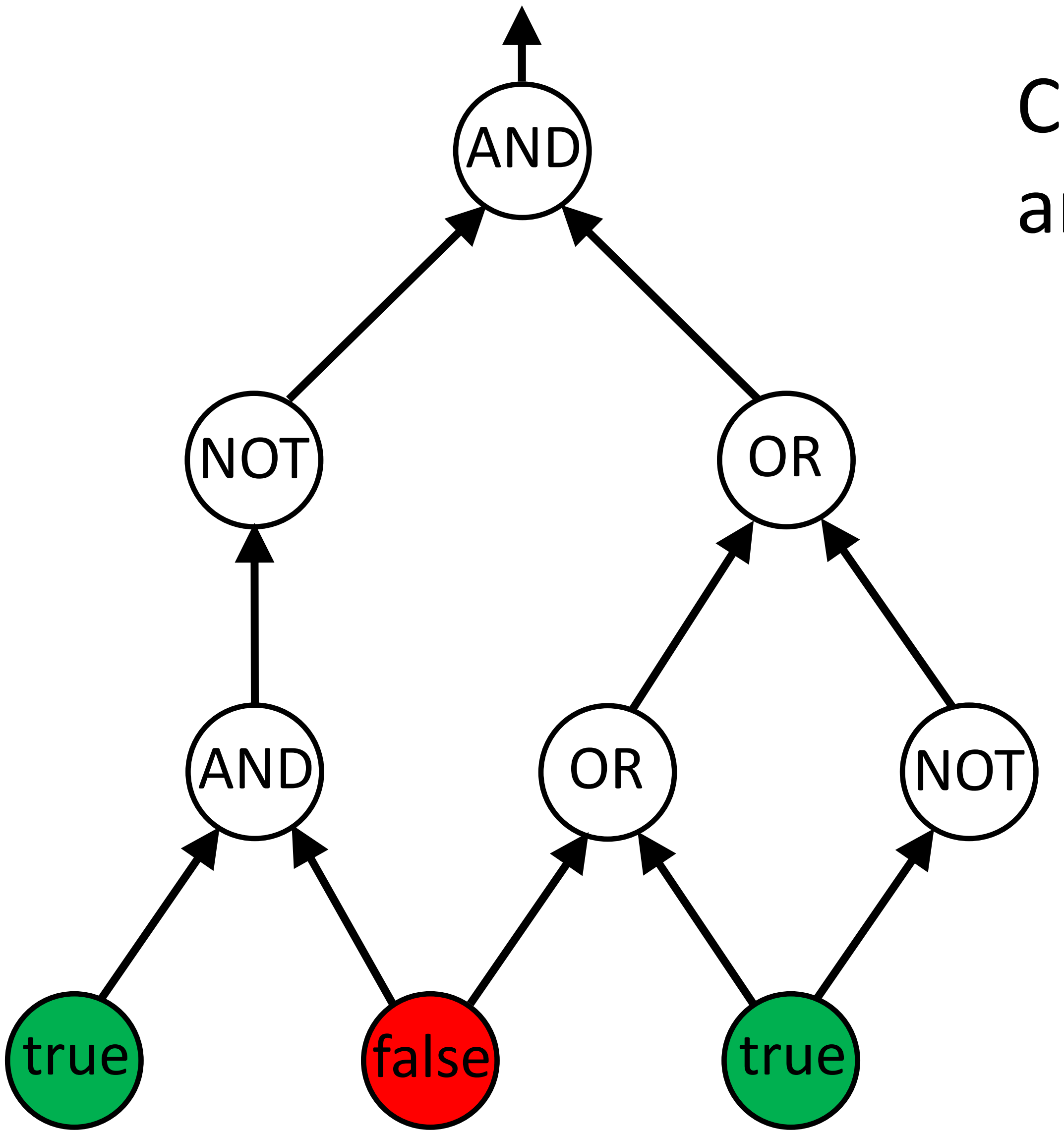
Test 2 Logistics

1. During class on Thursday 10/23.
2. You can bring your book and any notes you would like, but no electronic devices.
3. You may assume anything proven in class or on homework.
4. Three questions (13 points + 1 bonus):
    1) Integer optimal solutions (3 points).
    2) Duality (5 points).
    3) Solve a problem (5 points).
    4) Bonus (1 point).



Circuit Value Problem: When the gates are evaluated, is the output true?

Given a circuit schematic, what is the answer?



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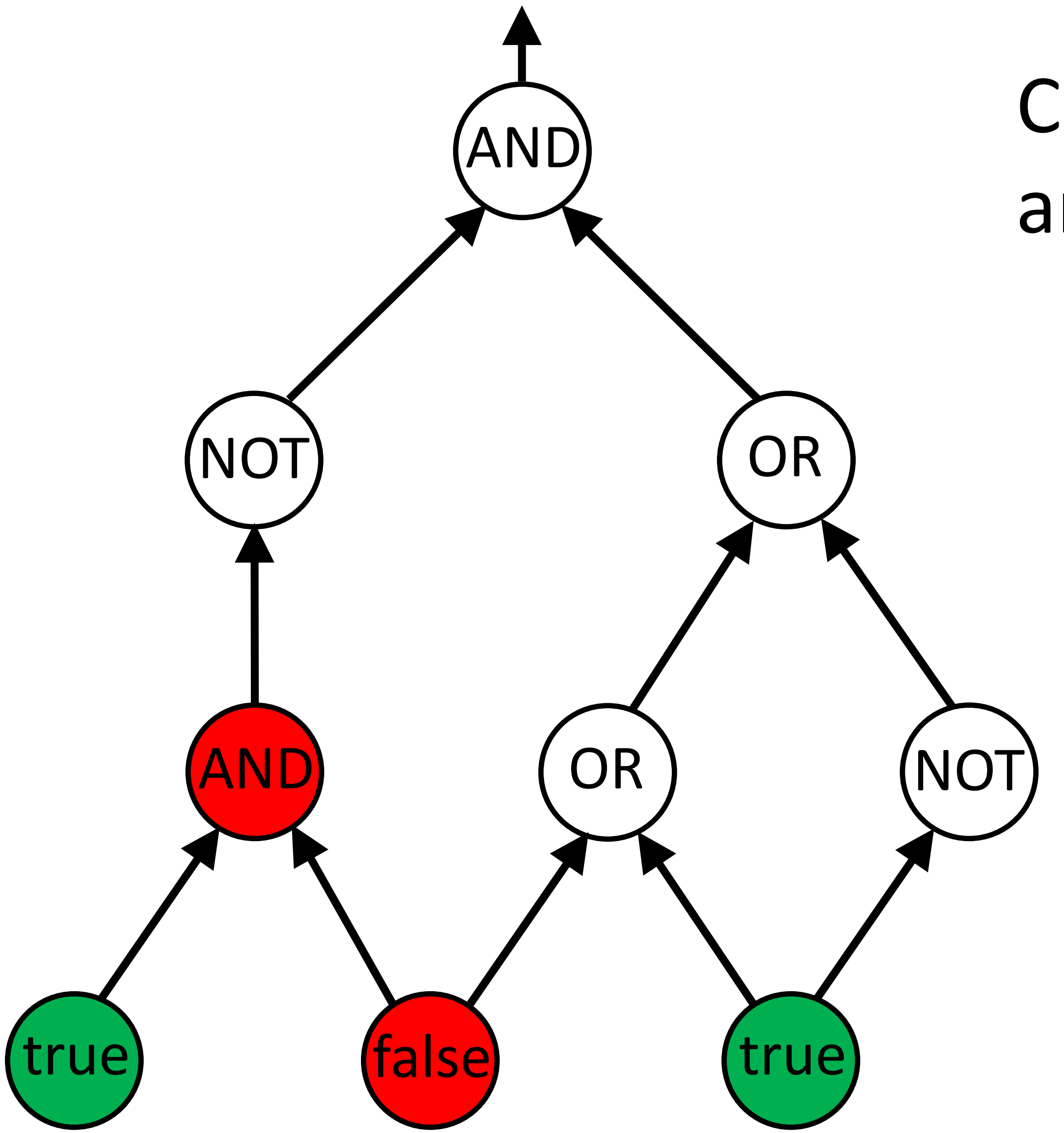

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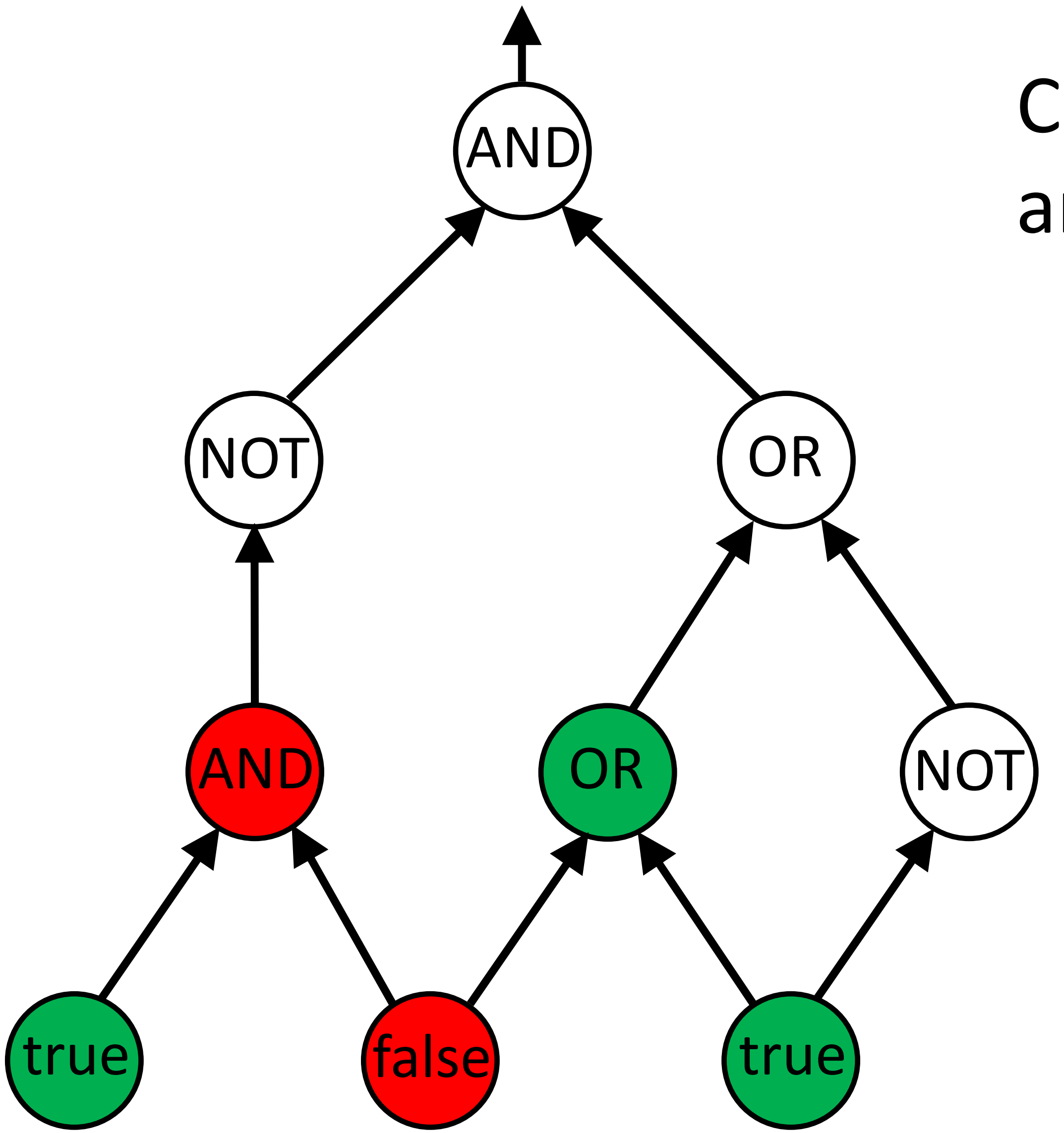

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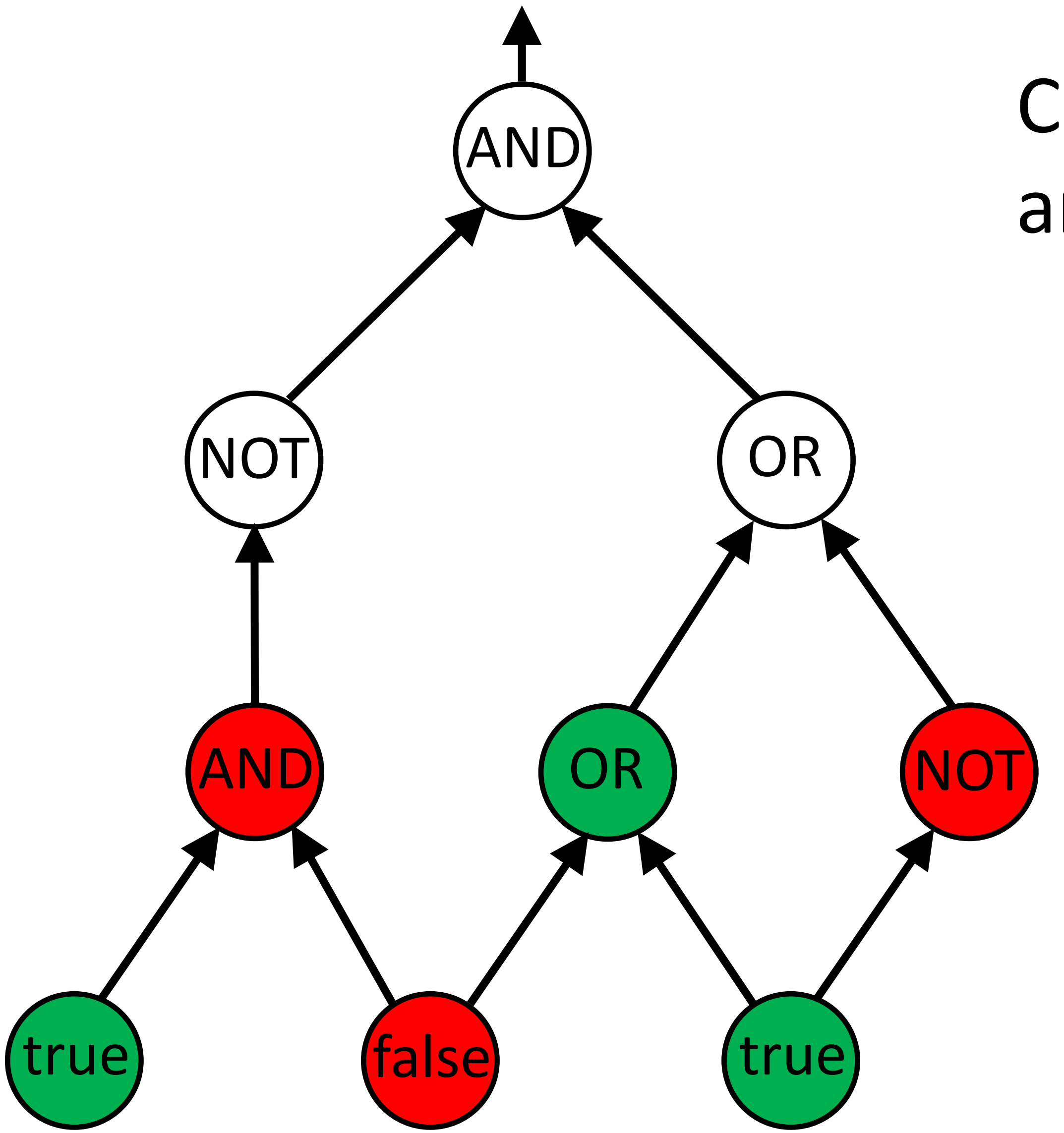

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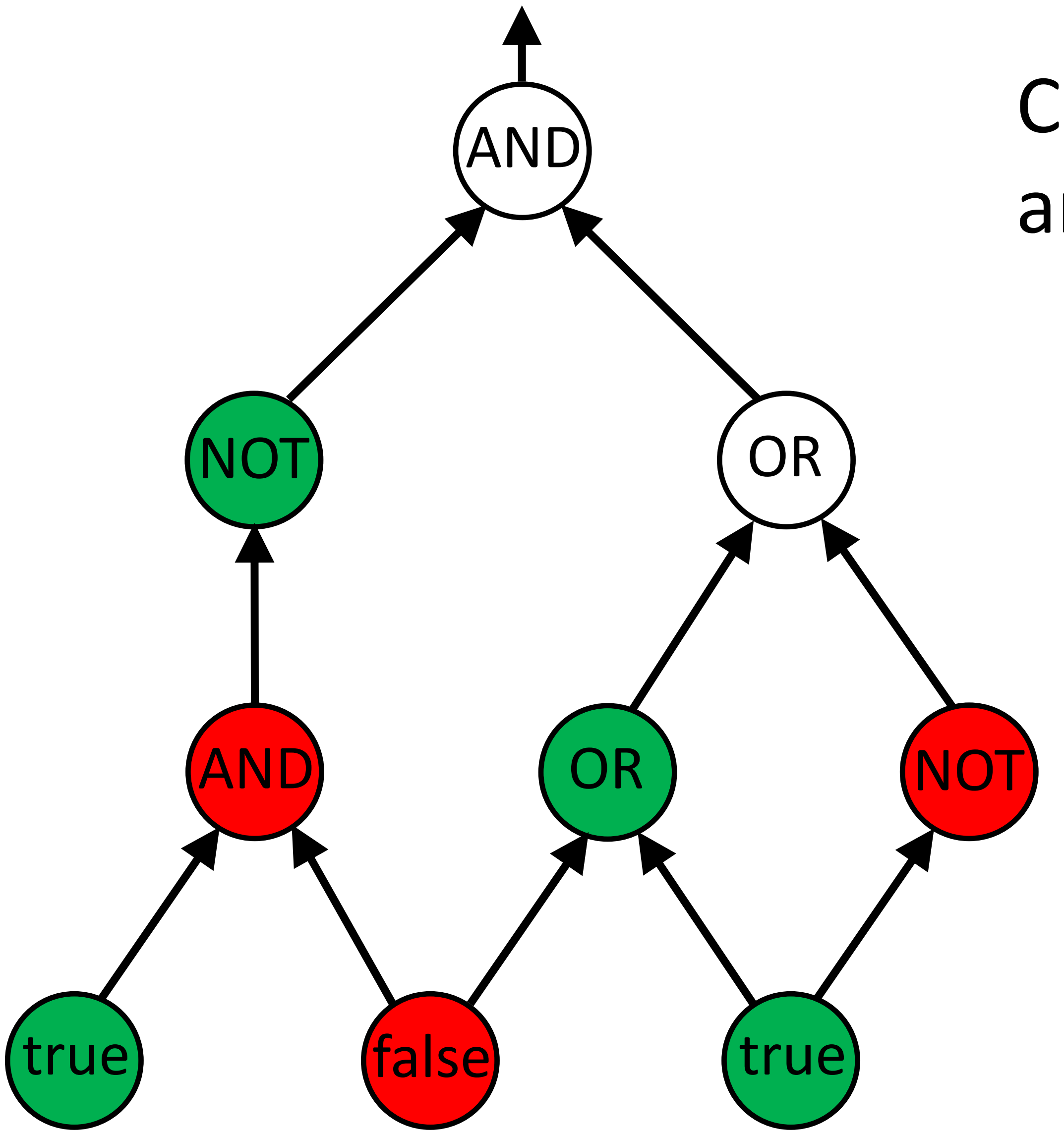
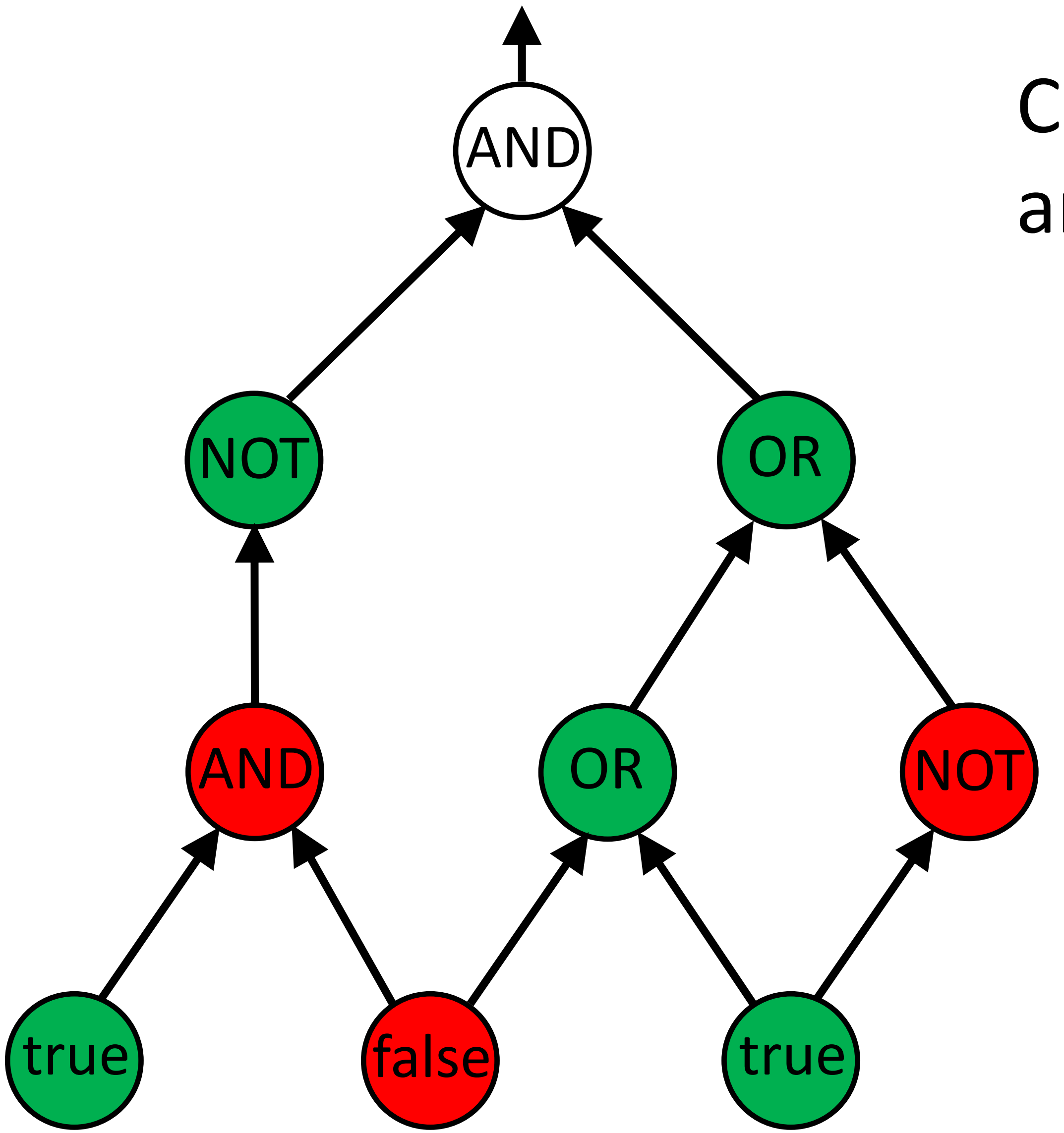


Circuit Value Problem: When the gates are evaluated, is the output true?

Given a circuit schematic, what is the answer?



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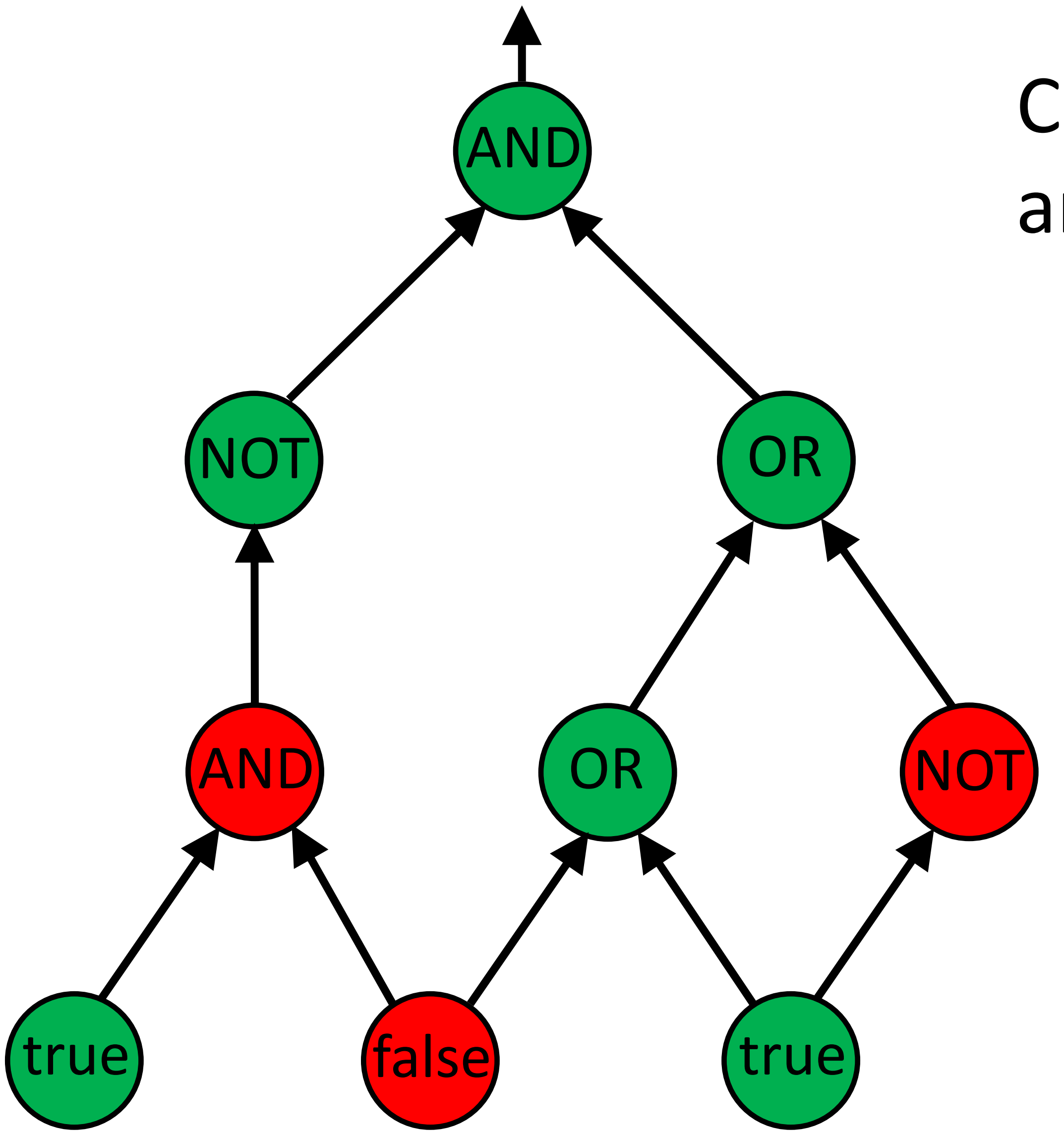

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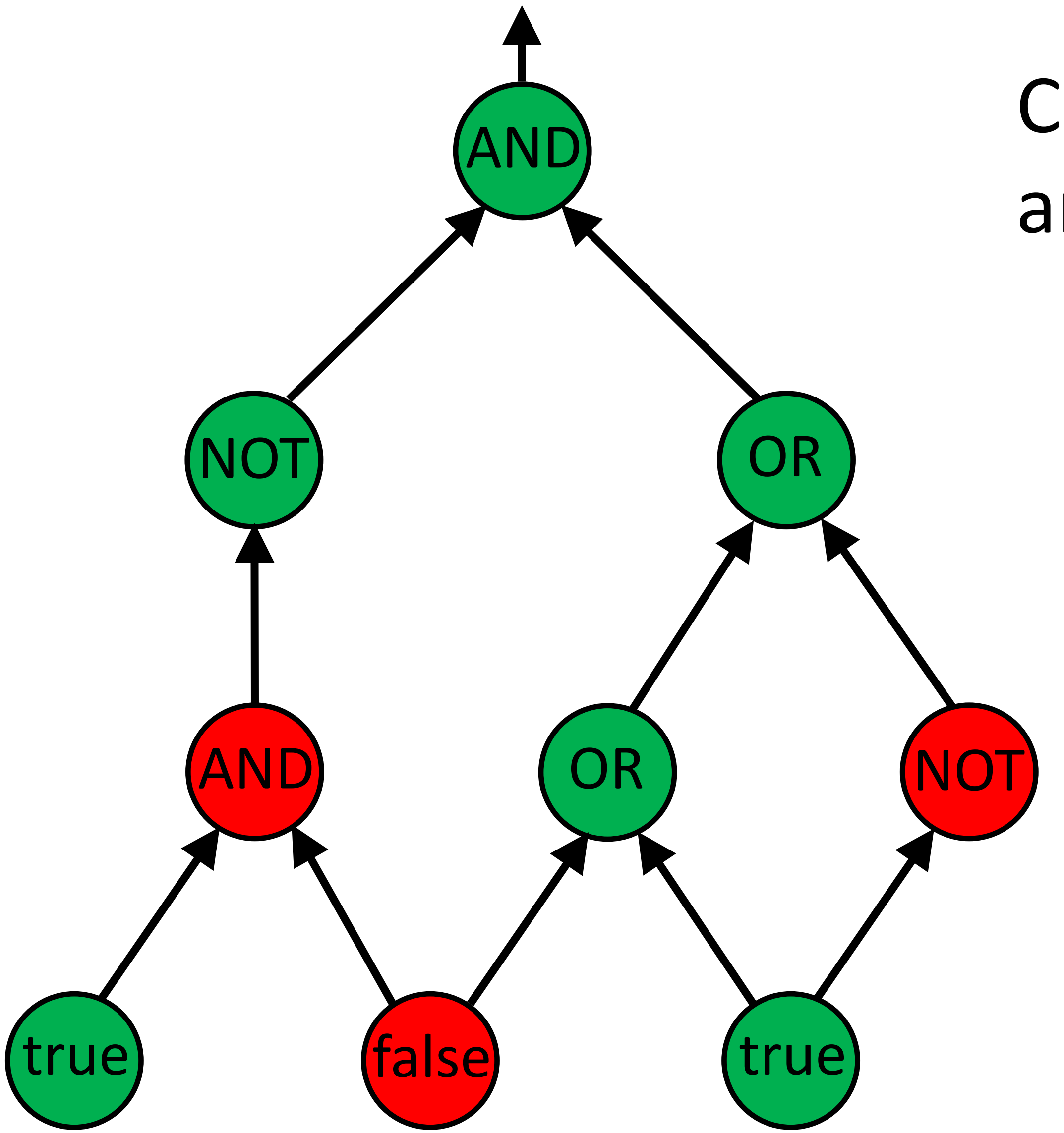
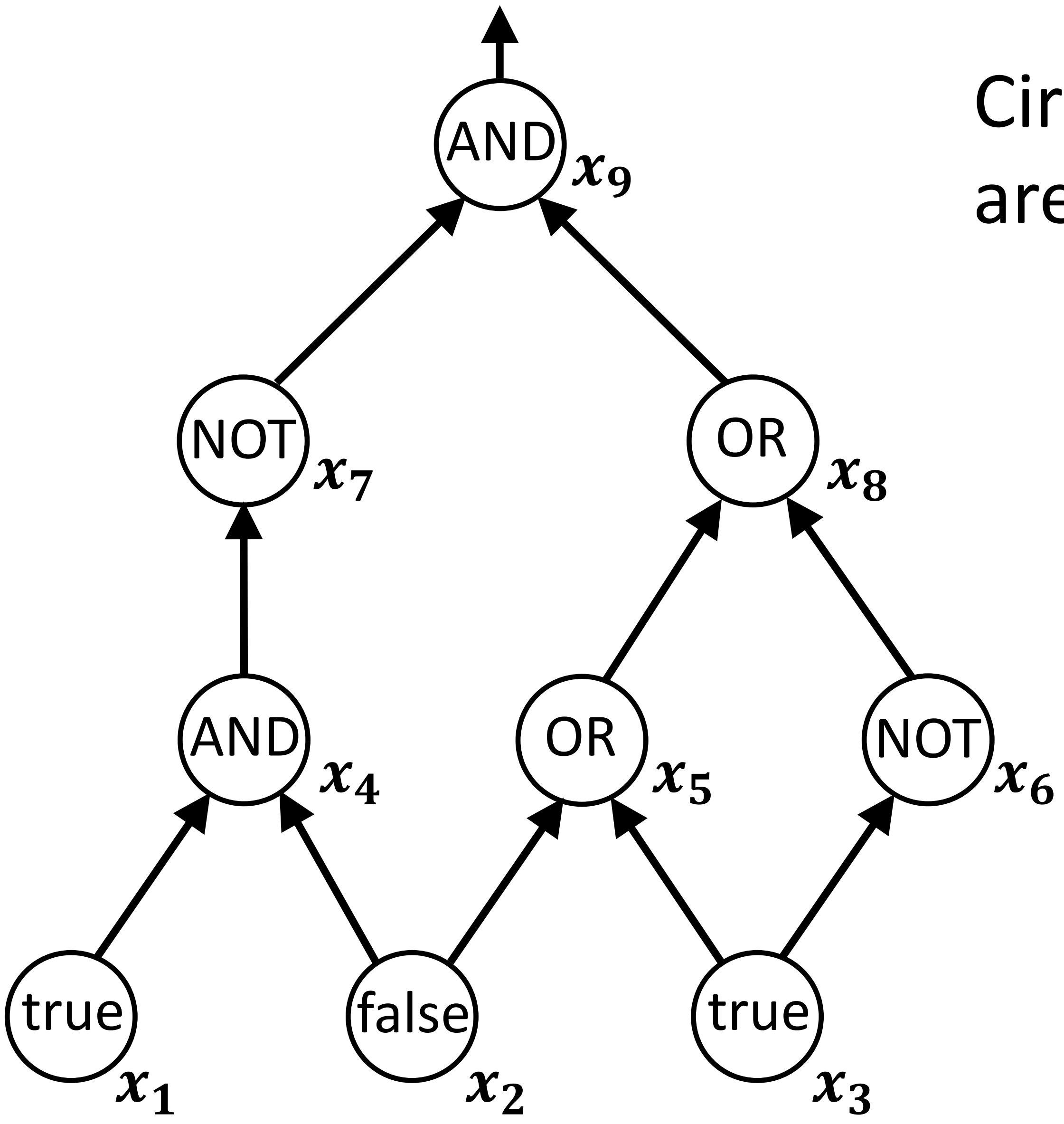


Circuit Value Problem: When the gates are evaluated, is the output true?

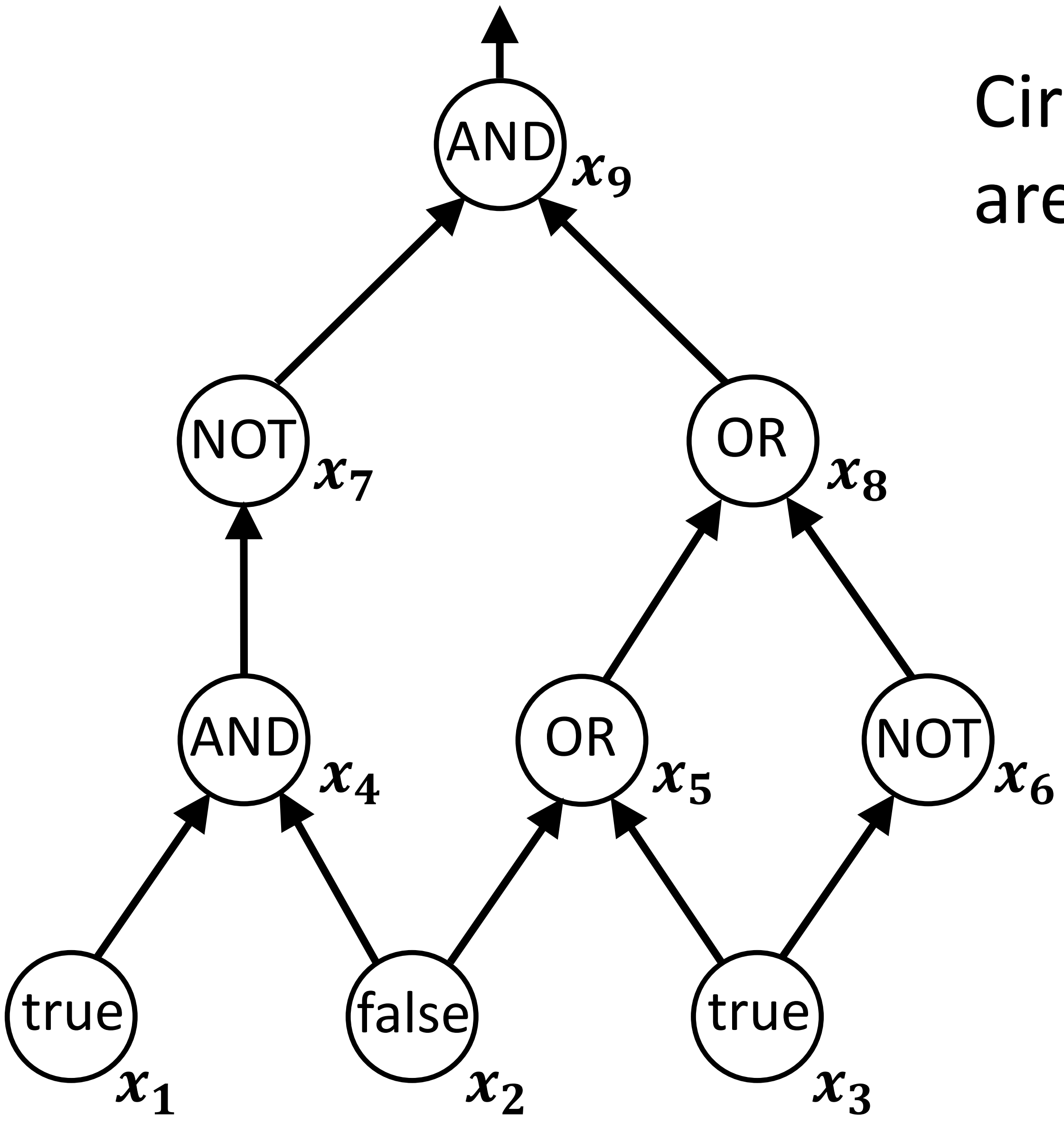
Given a circuit schematic, what is the answer?

Can we make a Linear Program that solves this?



Circuit Value Problem: When the gates are evaluated, is the output true?

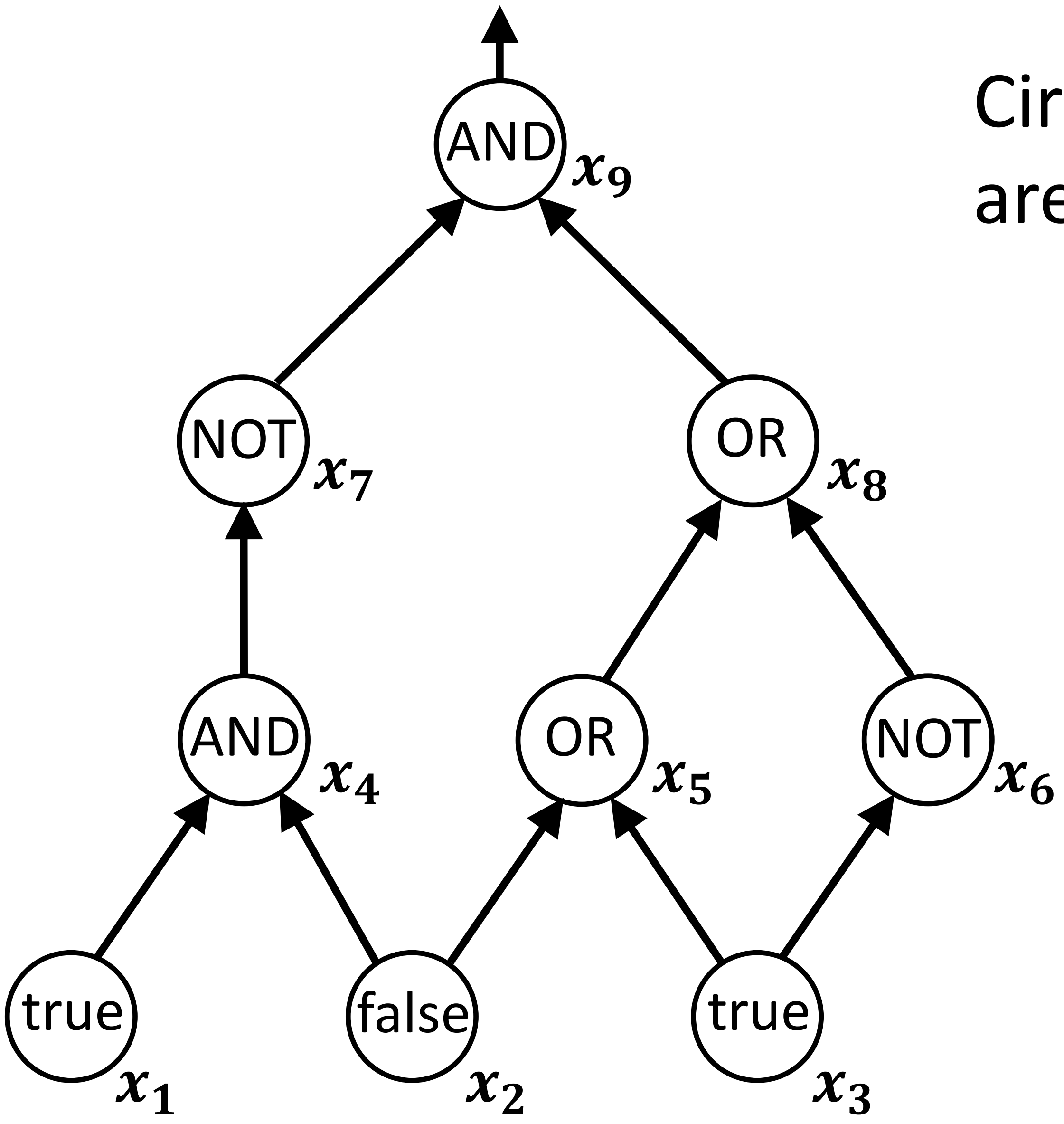
x_i = gate's evaluated value



Circuit Value Problem: When the gates are evaluated, is the output true?

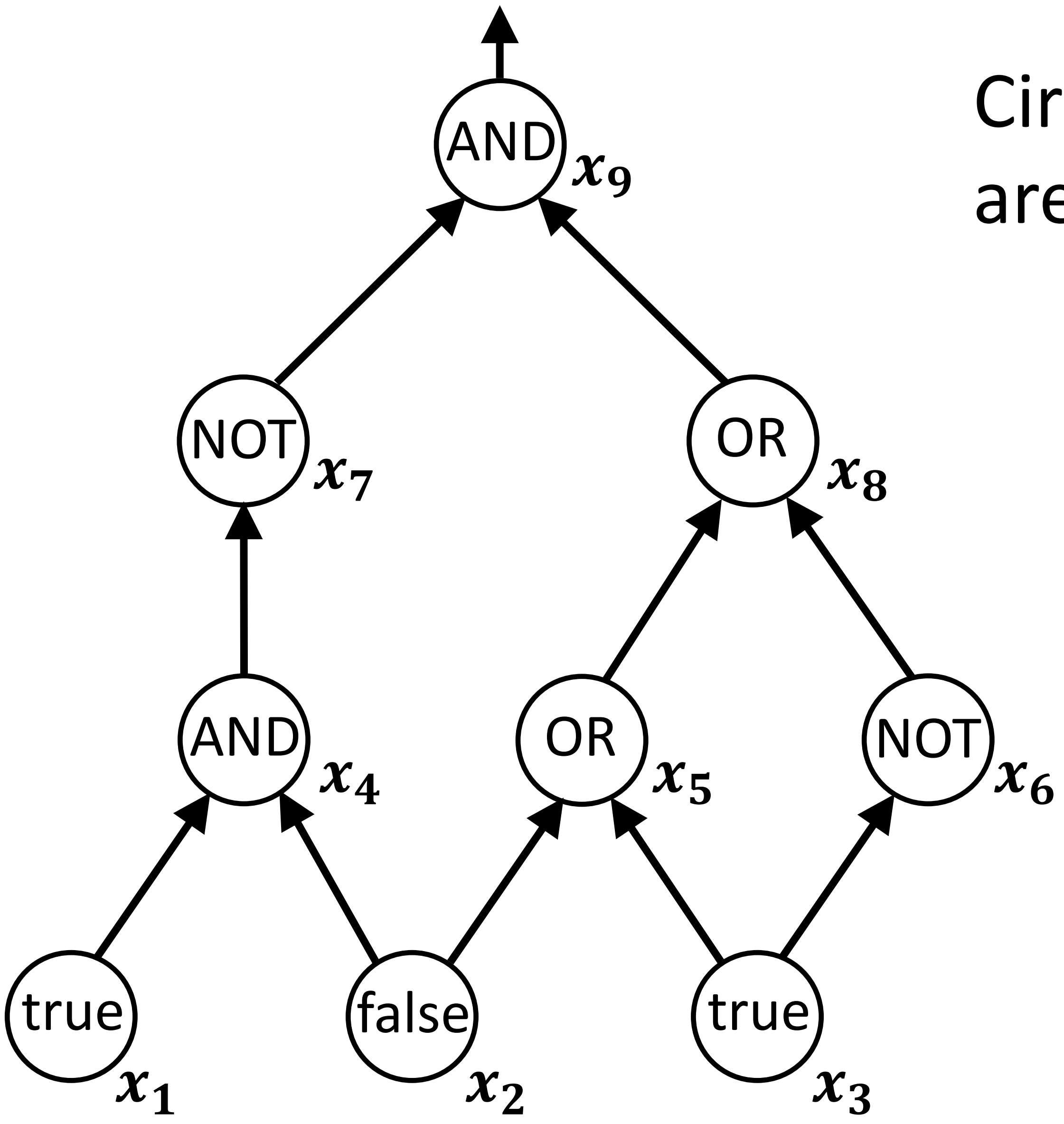
x_i = gate's evaluated value

What is x_9's value?



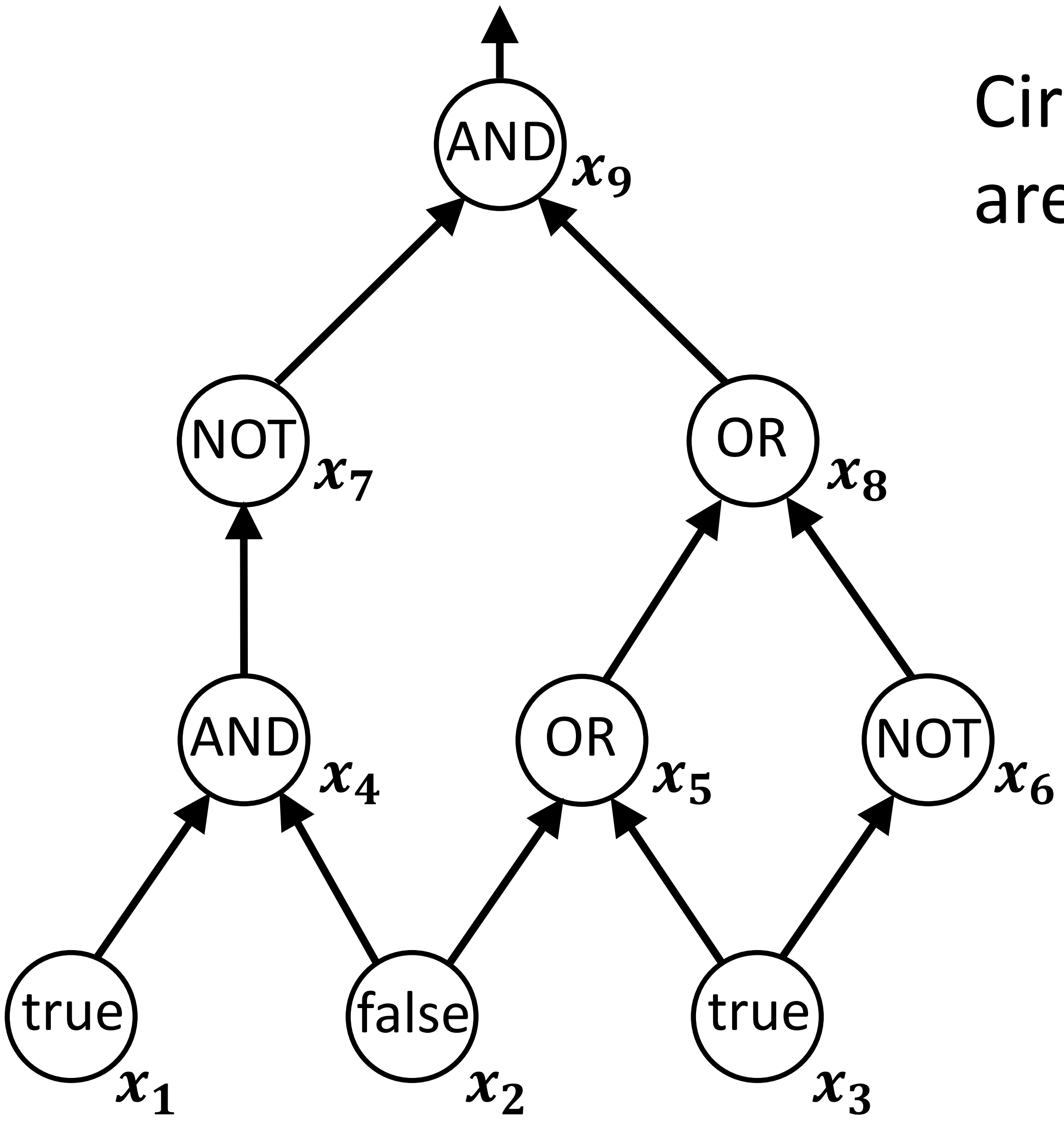
Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value
Objective: ??



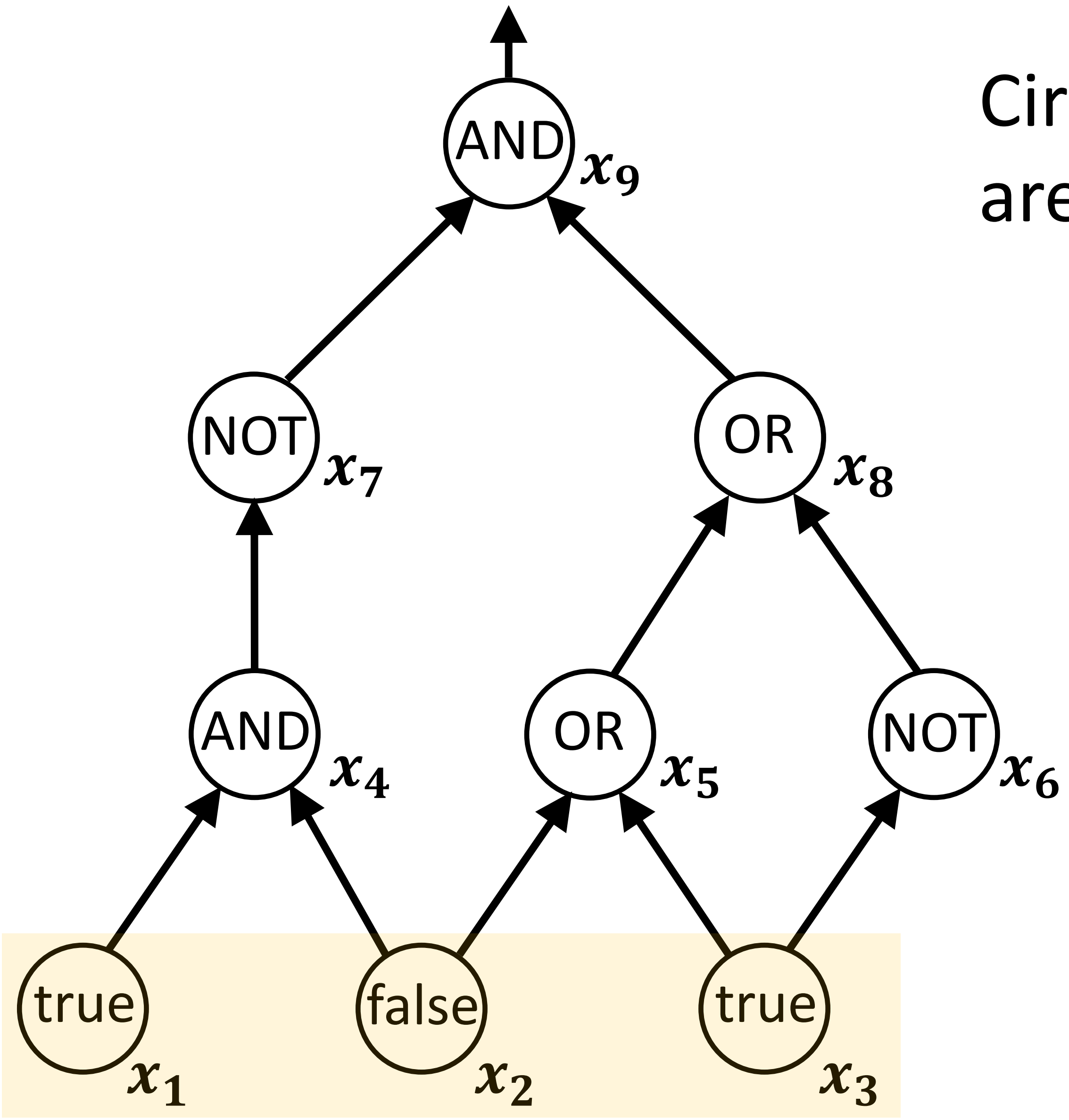
Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value
Objective: ??
Subject to:



Circuit Value Problem: When the gates are evaluated, is the output true?

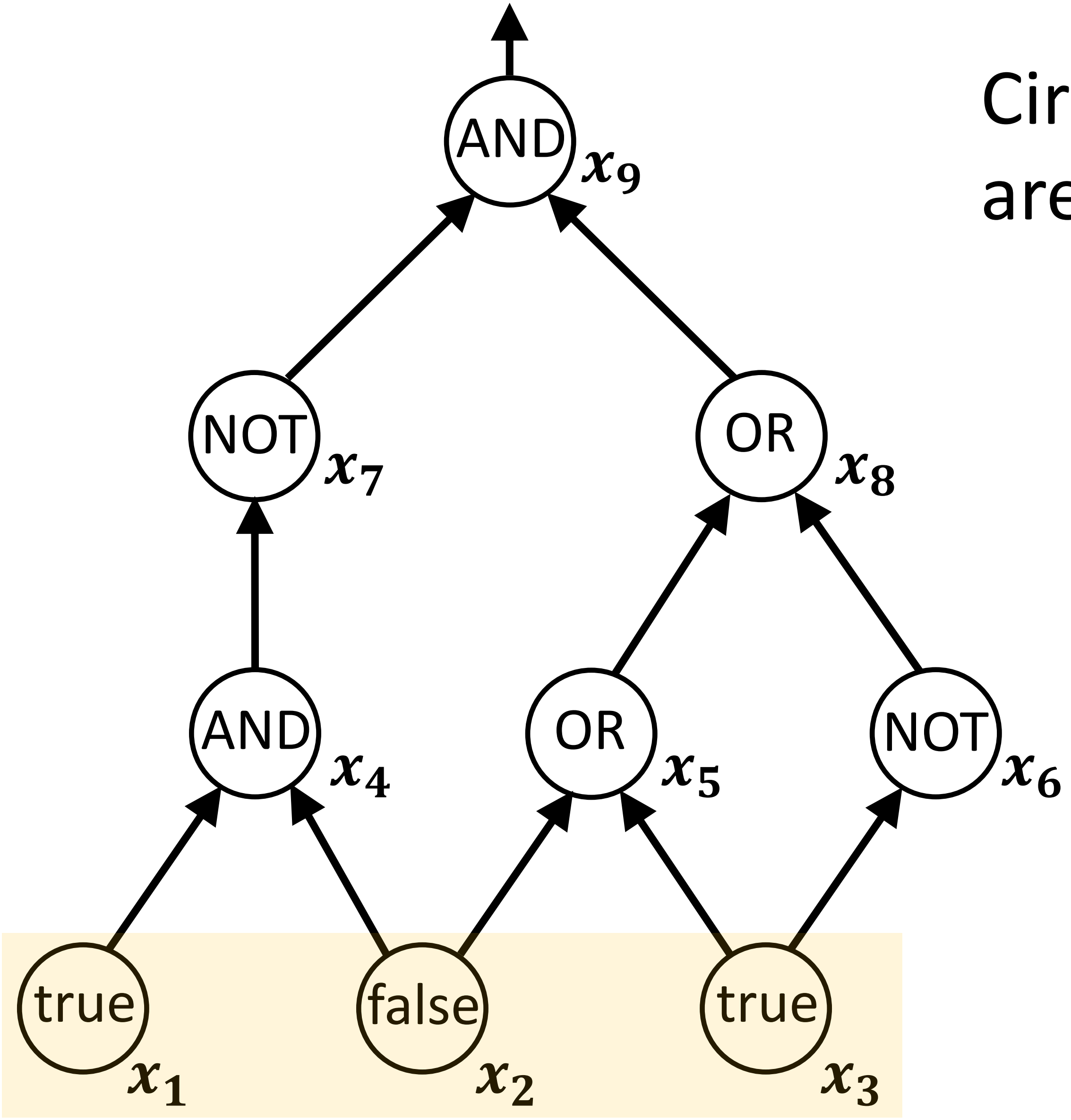
x_i = gate's evaluated value
Objective: ??
Subject to: $x_i \in \{0,1\}, \forall i$



Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value
Objective: ??
Subject to: $x_i \in \{0,1\}, \forall i$

$x_1 = ?$
$x_2 = ?$
$x_3 = ?$



Circuit Value Problem: When the gates are evaluated, is the output true?

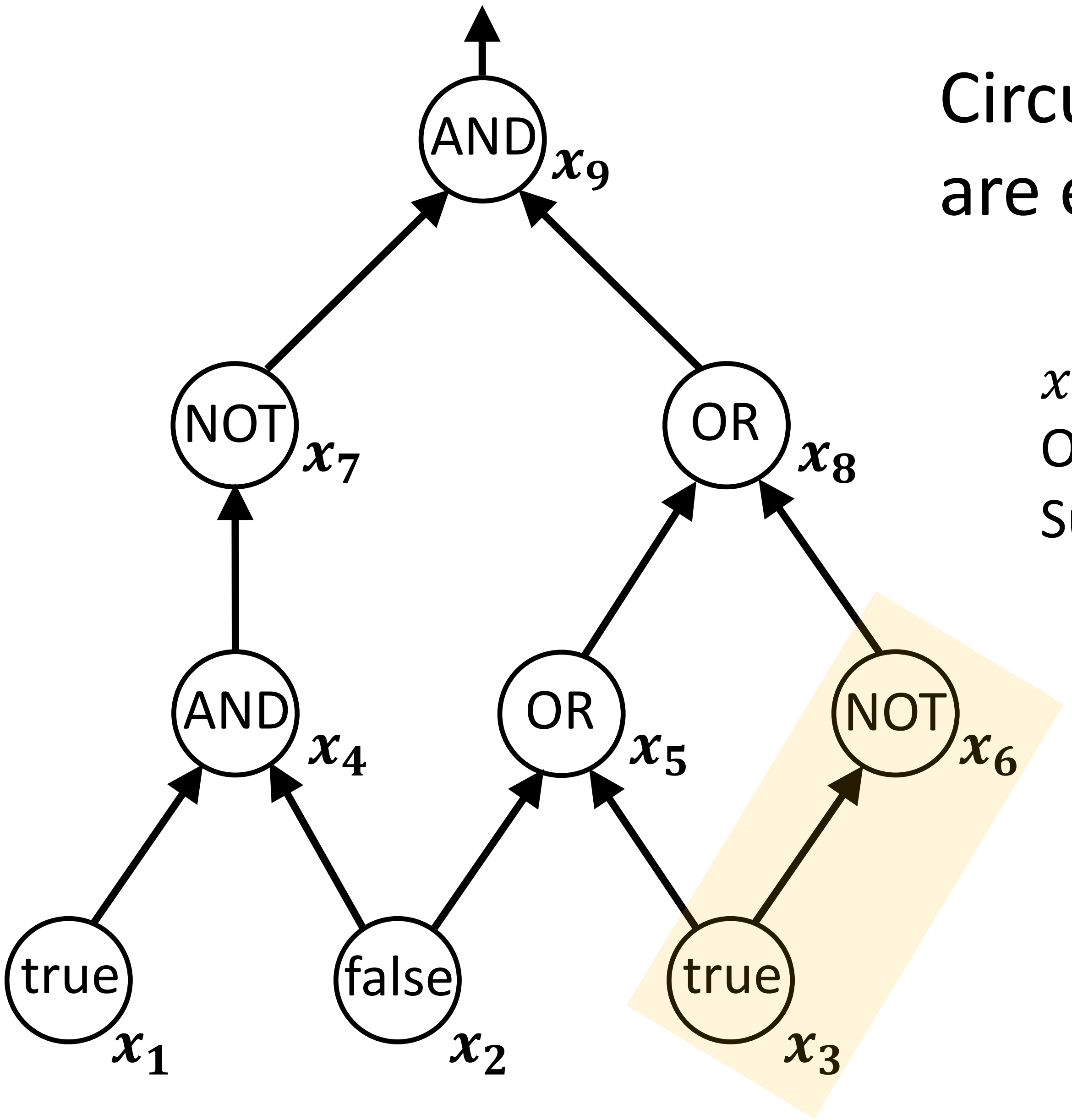
x_i = gate's evaluated value

Objective: ??

Subject to: $x_i \in \{0,1\}, \forall i$

$x_1 = 1$
$x_2 = 0$
$x_3 = 1$

In general, if gate i is initialized to true (or false), make $x_i = 1$ (or 0).



Circuit Value Problem: When the gates are evaluated, is the output true?

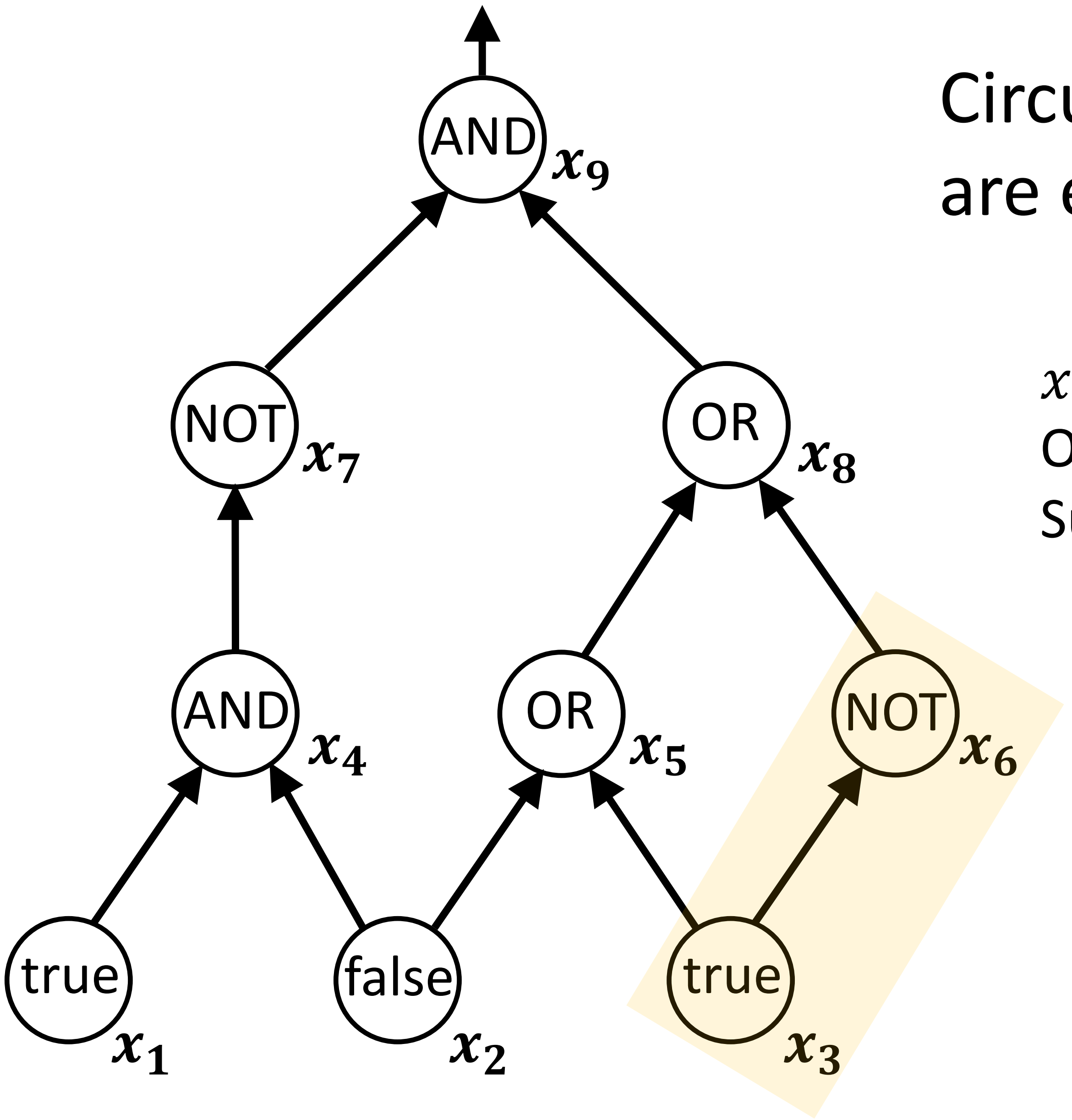
x_i = gate's evaluated value

Objective: ??

Subject to: $x_i \in \{0,1\}, \forall i$

$x_1 = 1$
$x_2 = 0$
$x_3 = 1$

$x_6 = ??$



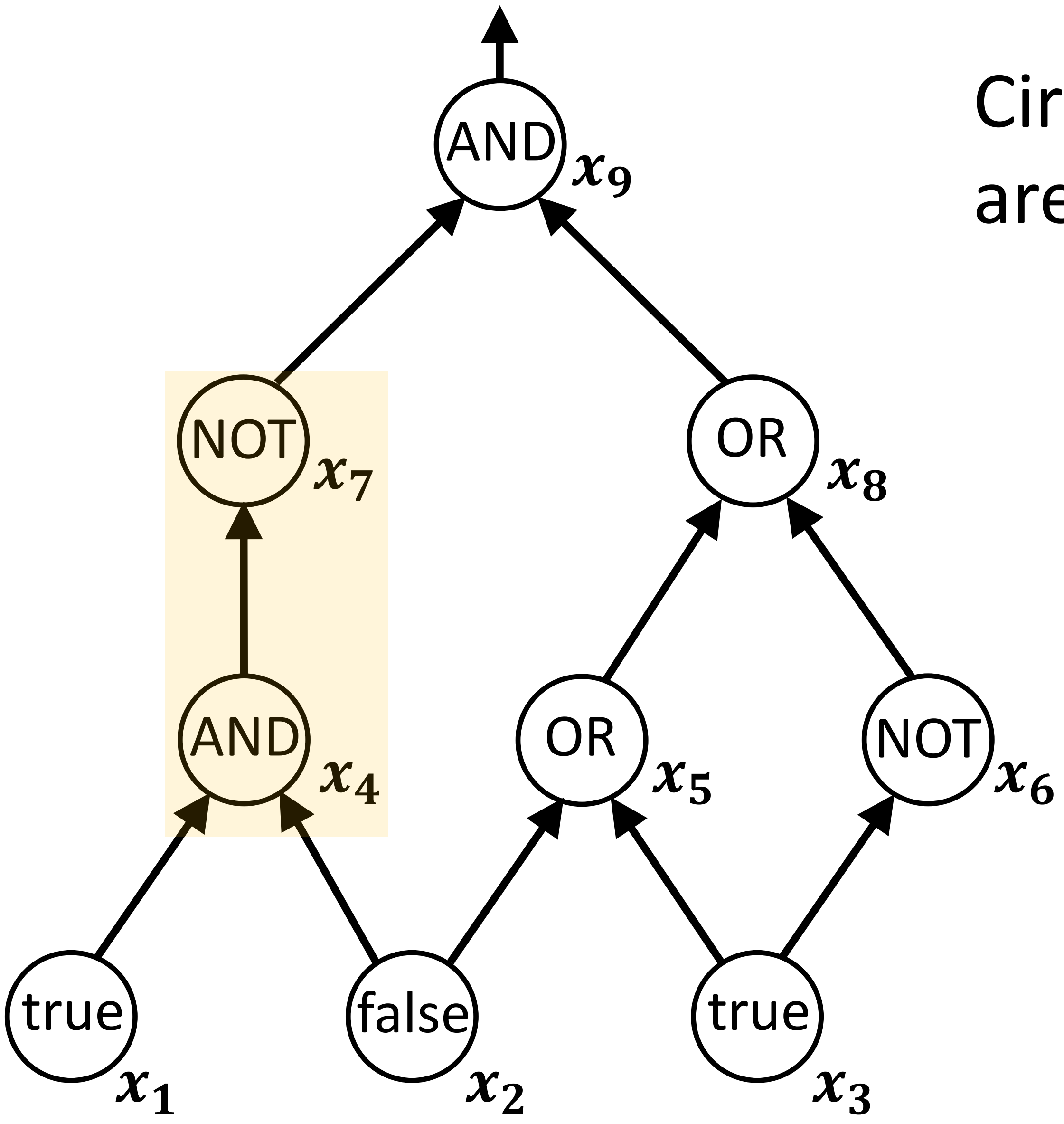
Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value
Objective: ??
Subject to: $x_i \in \{0,1\}, \forall i$

$x_1 = 1$
$x_2 = 0$
$x_3 = 1$

$x_6 = 1 - x_3$

In general, if gate $\boldsymbol{i}$ is NOT gate $\boldsymbol{j}$, make $\boldsymbol{x_i = 1 - x_j}$



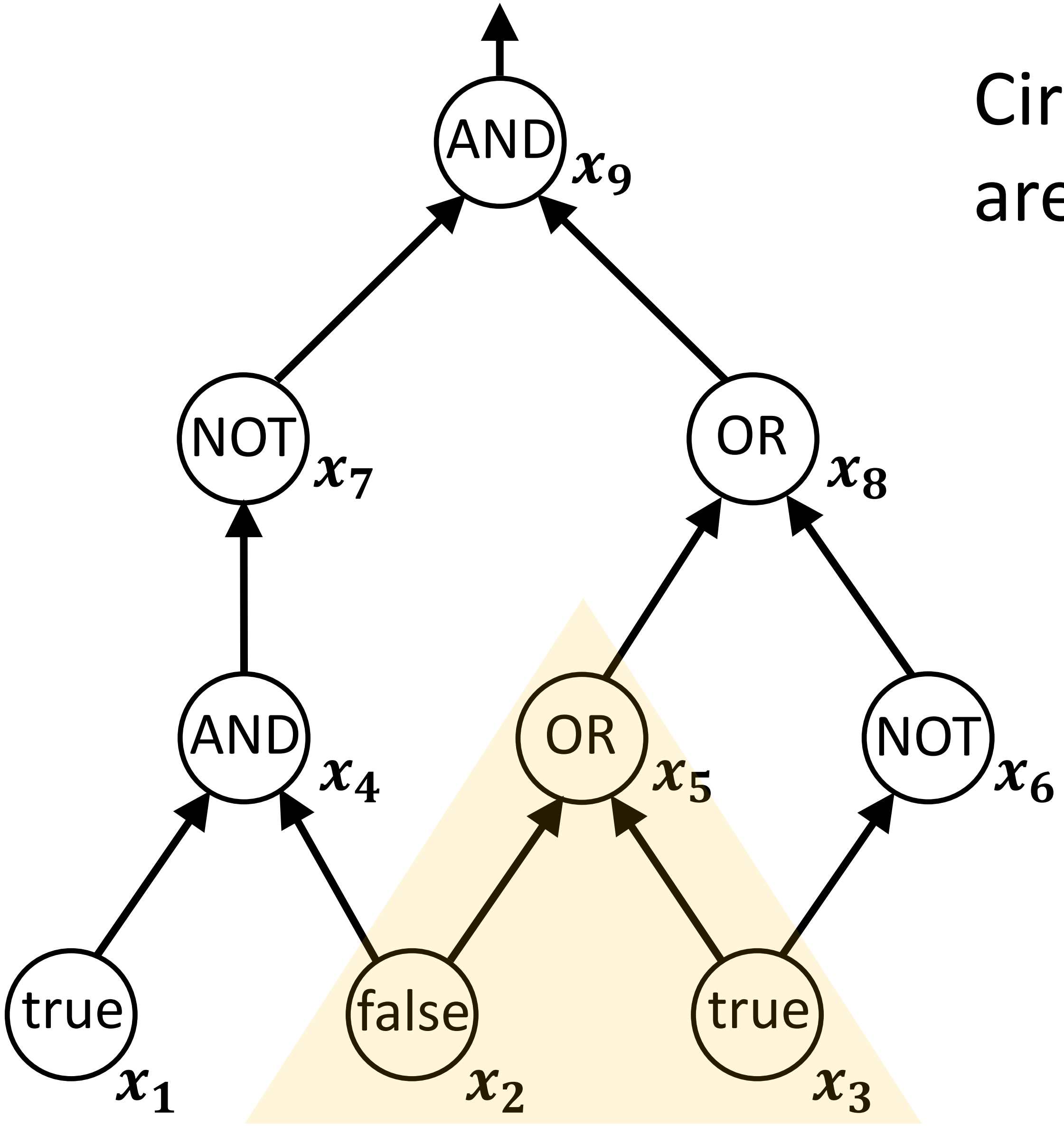
Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value

Objective: ??

Subject to: $x_i \in \{0,1\}, \forall i$

$$x_1 = 1$$
$$x_2 = 0$$
$$x_3 = 1$$
$$x_6 = 1 - x_3$$
$$x_7 = 1 - x_4$$



Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value

Objective: ??

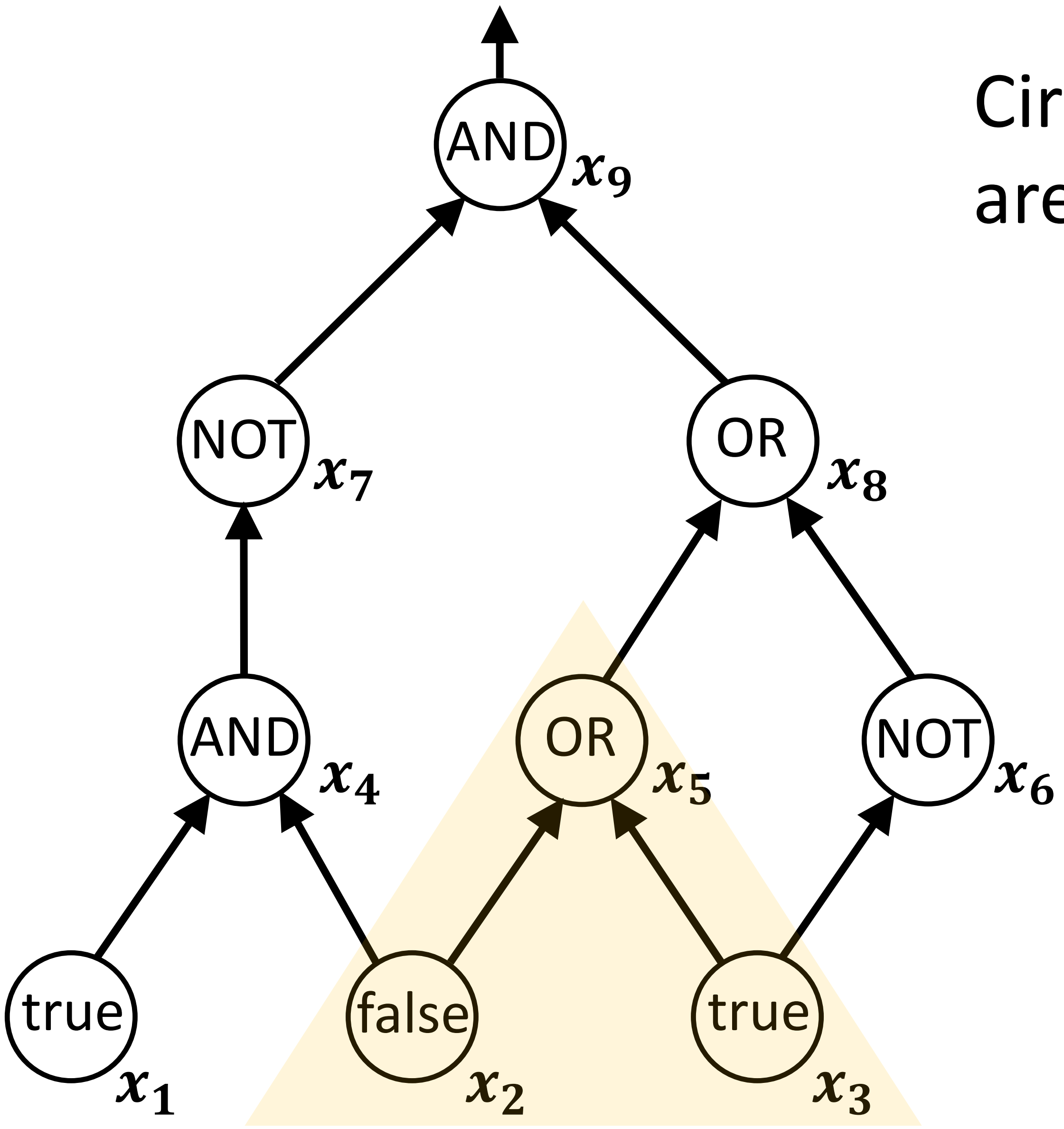
Subject to: $x_i \in \{0,1\}, \forall i$

$x_1 = 1$
$x_2 = 0$
$x_3 = 1$

$x_6 = 1 - x_3$

$x_7 = 1 - x_4$

??



Circuit Value Problem: When the gates are evaluated, is the output true?

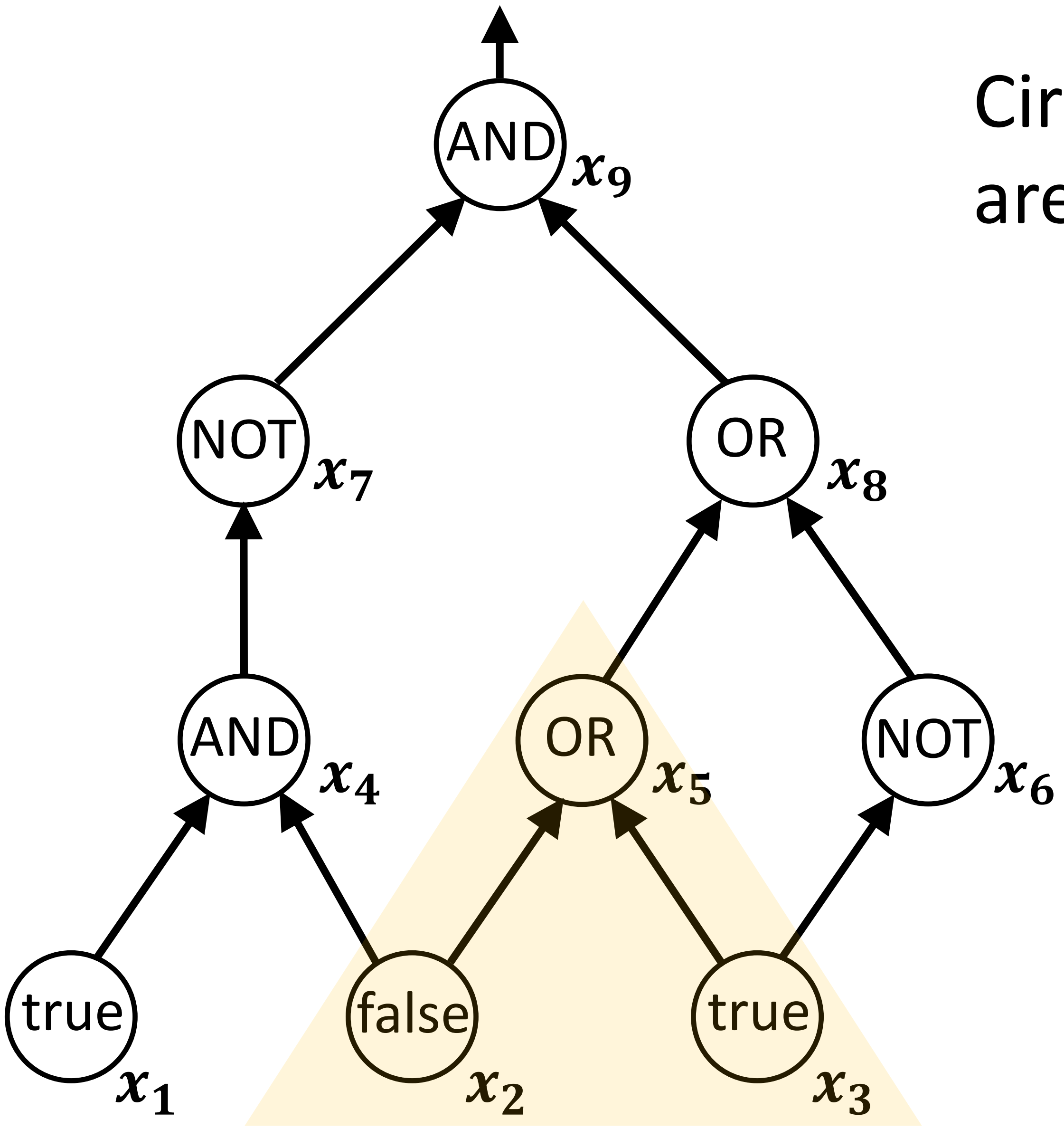
x_i = gate's evaluated value

Objective: ??

Subject to: $x_i \in \{0,1\}, \forall i$

$$x_1 = 1$$
$$x_2 = 0$$
$$x_3 = 1$$
$$x_6 = 1 - x_3$$
$$x_7 = 1 - x_4$$

x_j	x_k	$x_i = x_j$ OR x_k
0	0	0
1	0	1
0	1	1
1	1	1



Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value
Objective: ??
Subject to: $x_i \in \{0,1\}, \forall i$

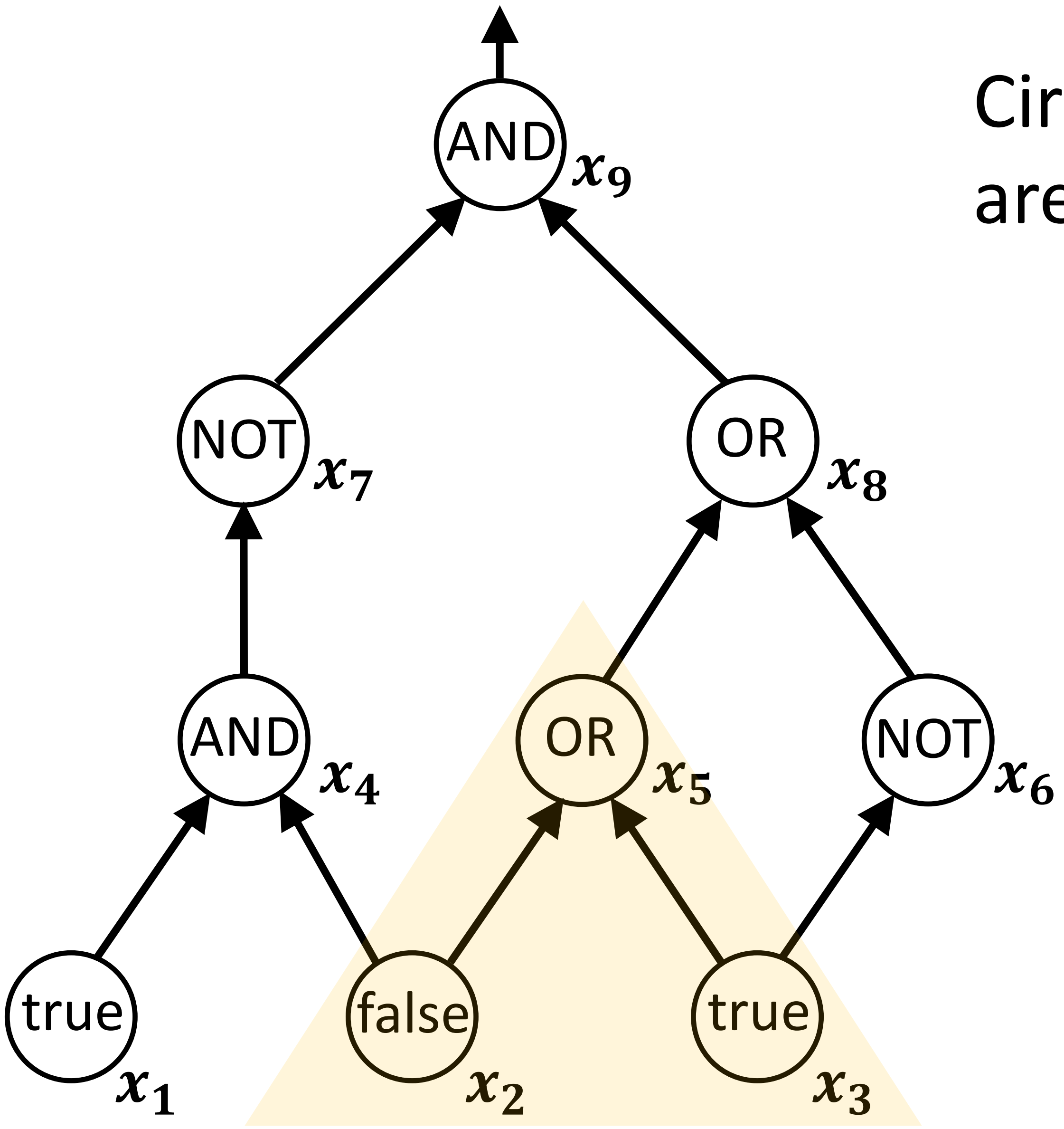
$x_1 = 1$
$x_2 = 0$
$x_3 = 1$

$x_6 = 1 - x_3$

$x_7 = 1 - x_4$

$x_5 \geq x_2$

x_j	x_k	$x_i = x_j$ OR x_k
0	0	0
1	0	1
0	1	1
1	1	1



Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value

Objective: ??

Subject to: $x_i \in \{0,1\}, \forall i$

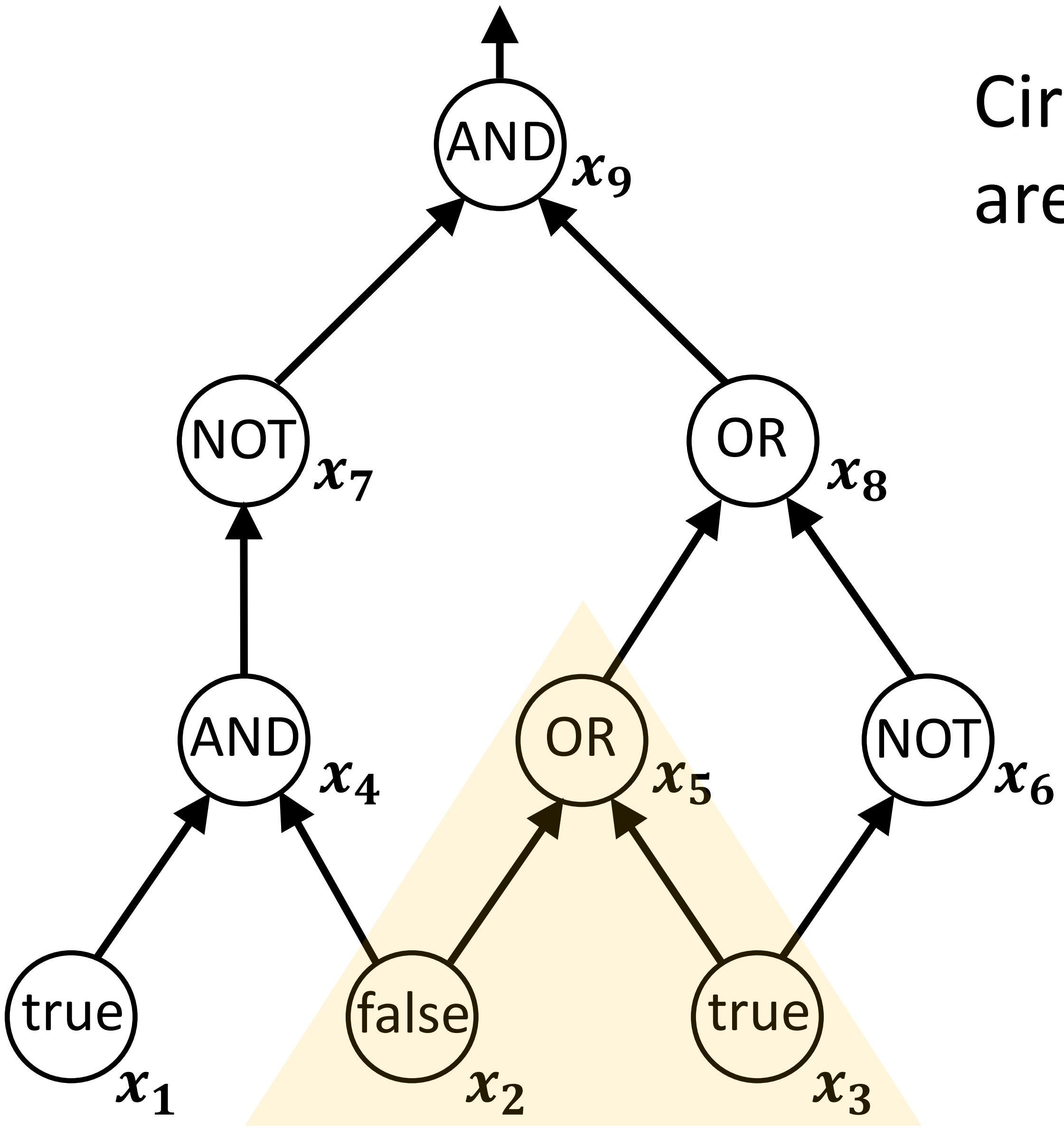
$$x_1 = 1$$
$$x_2 = 0$$
$$x_3 = 1$$

$$x_6 = 1 - x_3$$

$$x_7 = 1 - x_4$$

$$x_5 \geq x_2$$
$$x_5 \geq x_3$$

x_j	x_k	$x_i = x_j$ OR x_k
0	0	0
1	0	1
0	1	1
1	1	1



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$x_2 = 0$
$x_3 = 1$

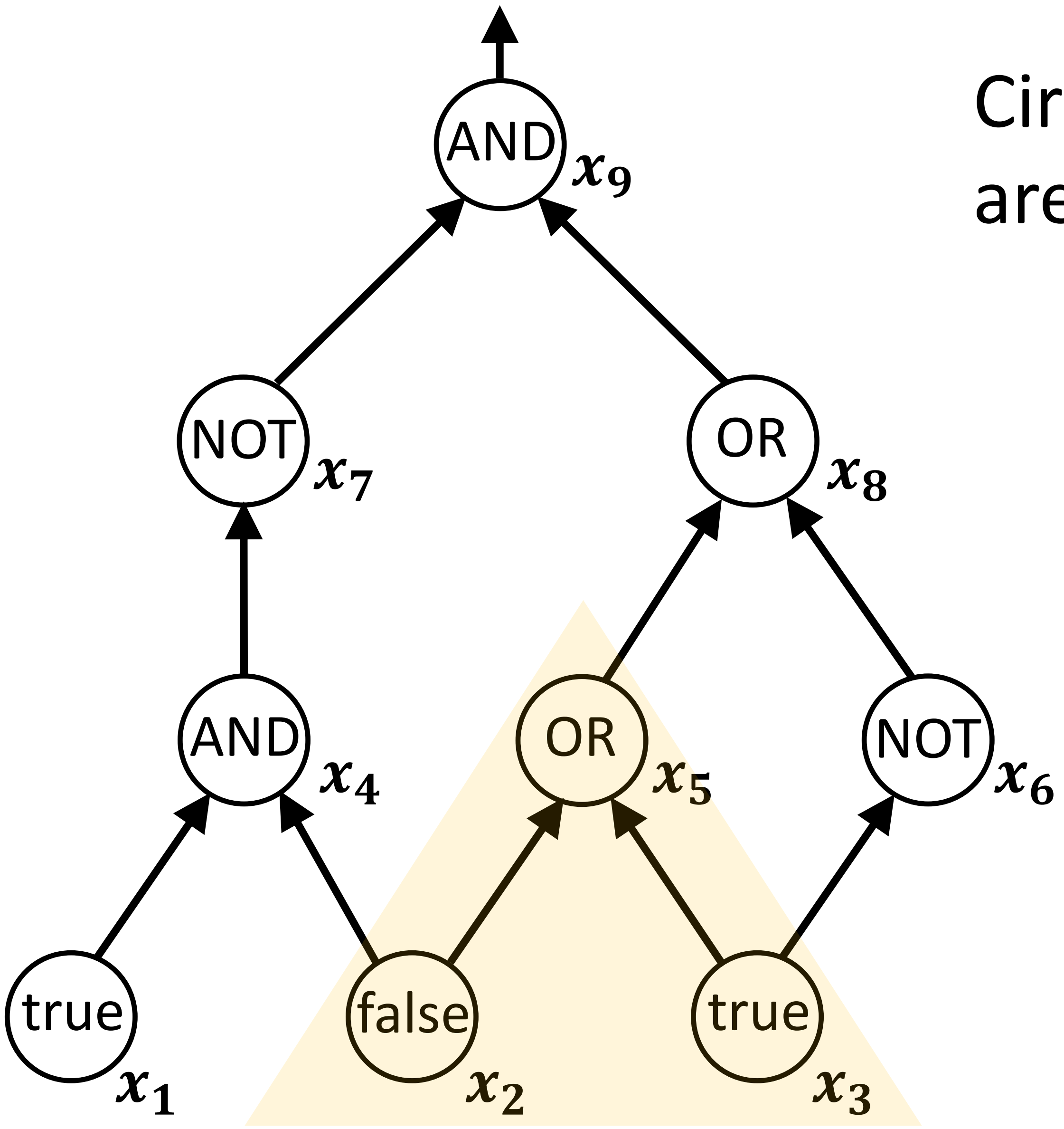
$x_6 = 1 - x_3$

$x_7 = 1 - x_4$

$x_5 \geq x_2$
$x_5 \geq x_3$

x_j	x_k	$x_i = x_j$ OR x_k
0	0	0
1	0	1
0	1	1
1	1	1

Is that enough?



Circuit Value Problem: When the gates are evaluated, is the output true?

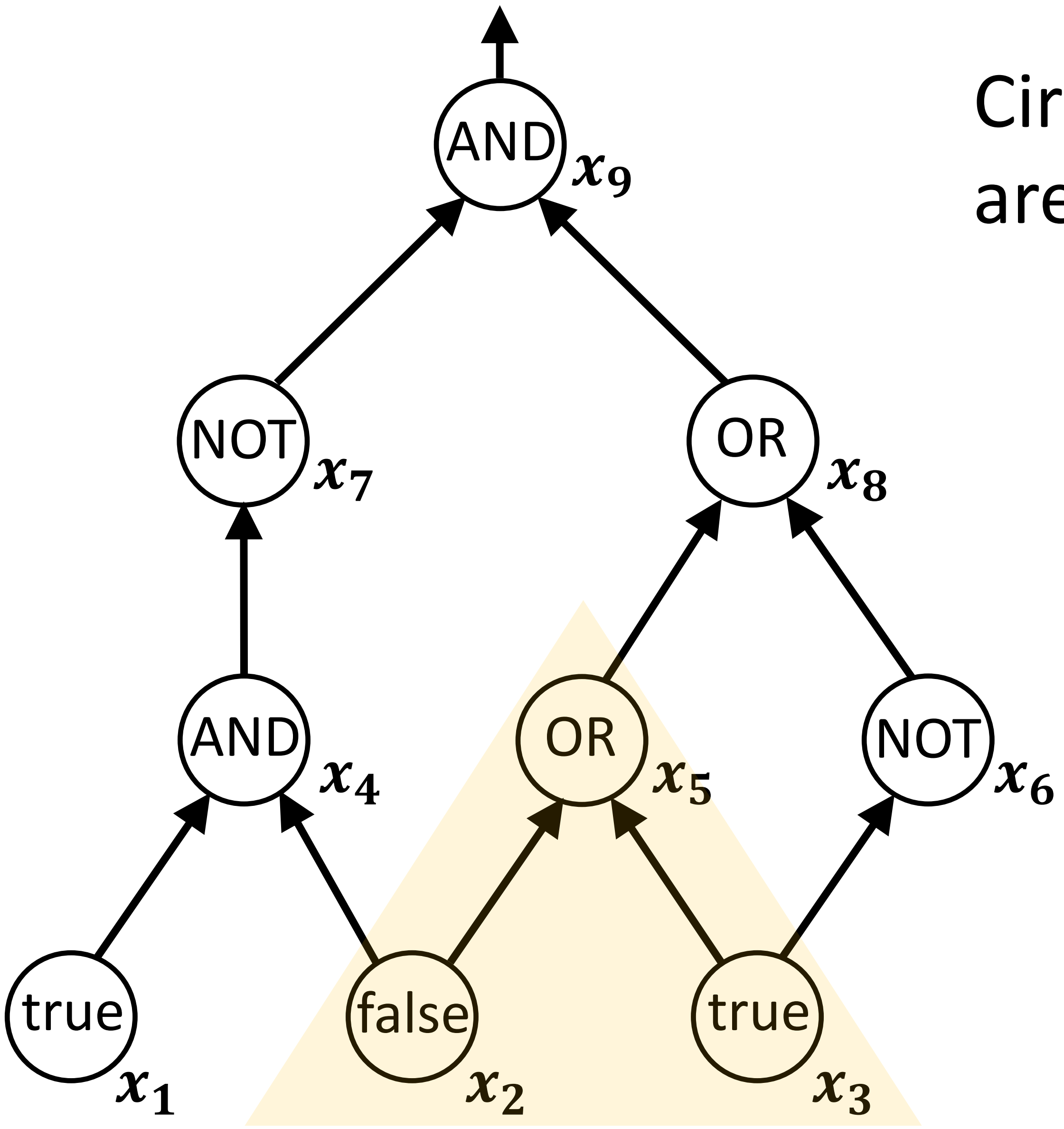
x_i = gate's evaluated value

Objective: ??

Subject to: $x_i \in \{0,1\}, \forall i$

$$x_1 = 1$$
$$x_2 = 0$$
$$x_3 = 1$$
$$x_6 = 1 - x_3$$
$$x_7 = 1 - x_4$$
$$x_5 \geq x_2$$
$$x_5 \geq x_3$$
$$x_5 \leq x_2 + x_3$$

x_j	x_k	$x_i = x_j$ OR x_k
0	0	0
1	0	1
0	1	1
1	1	1



Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value

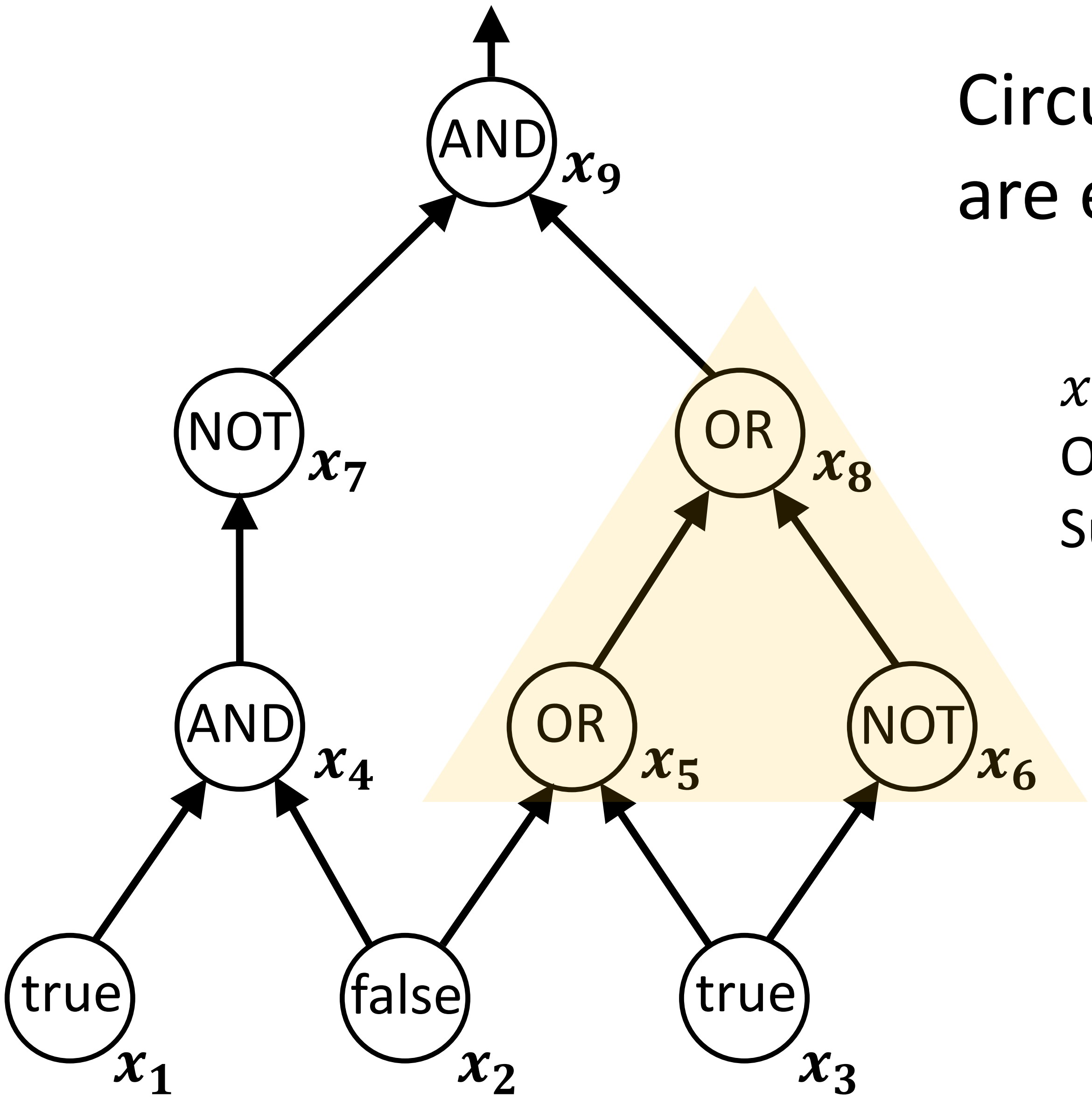
Objective: ??

Subject to: $x_i \in \{0,1\}, \forall i$

$$x_1 = 1$$
$$x_2 = 0$$
$$x_3 = 1$$
$$x_6 = 1 - x_3$$
$$x_7 = 1 - x_4$$
$$x_5 \geq x_2$$
$$x_5 \geq x_3$$
$$x_5 \leq x_2 + x_3$$

In general, if gate $\boldsymbol{i}$ is OR gates $\boldsymbol{j}, \boldsymbol{k}$:

$$\boldsymbol{x_i \geq x_j}$$
$$\boldsymbol{x_i \geq x_k}$$
$$\boldsymbol{x_i \leq x_j + x_k}$$



Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value

Objective: ??

Subject to: $x_i \in \{0,1\}, \forall i$

$$x_1 = 1$$
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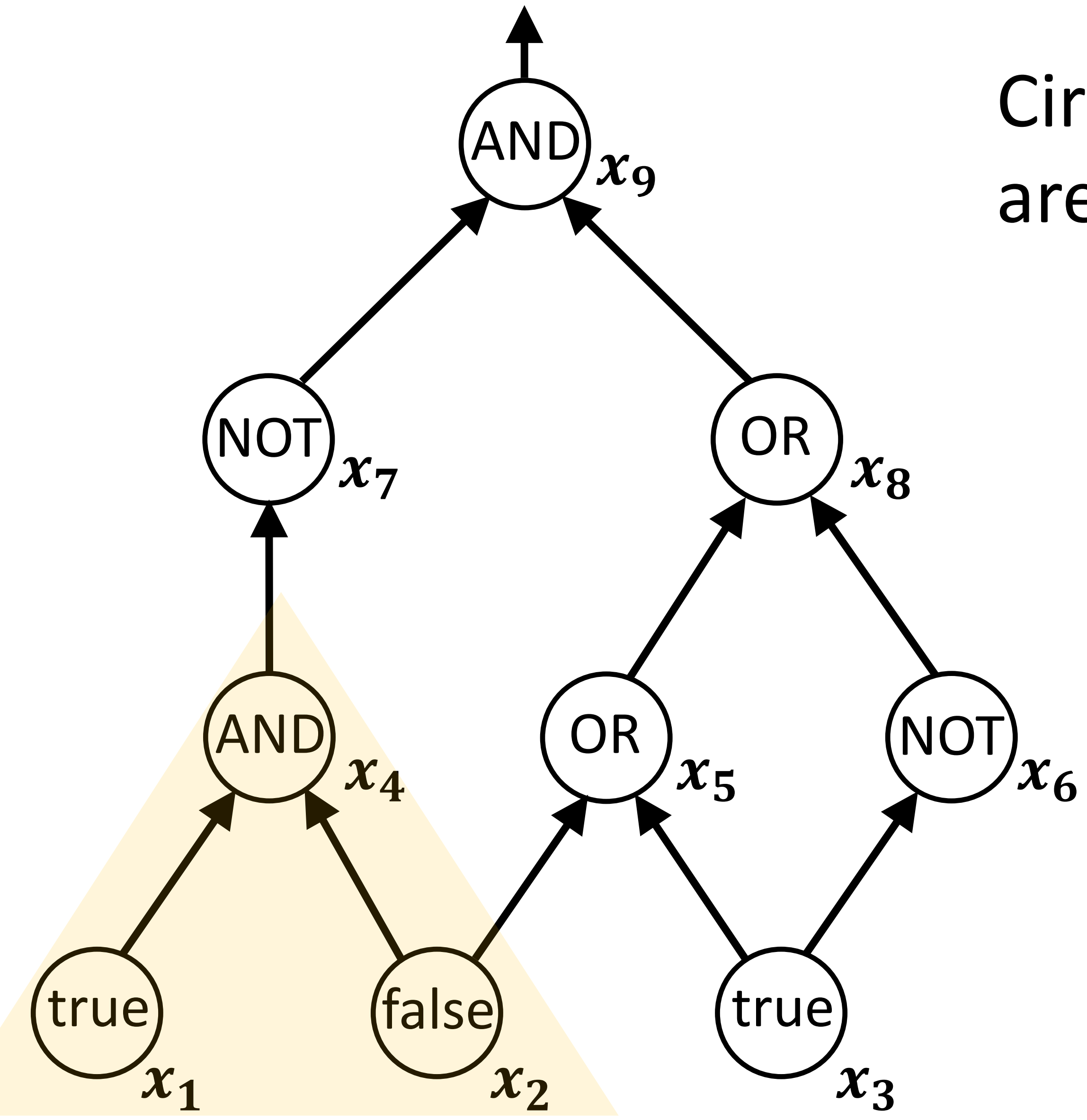
$$x_3 = 1$$

$$x_6 = 1 - x_3$$

$$x_7 = 1 - x_4$$

$$x_5 \geq x_2$$
$$x_5 \geq x_3$$
$$x_5 \leq x_2 + x_3$$

$$x_8 \geq x_5$$
$$x_8 \geq x_6$$
$$x_8 \leq x_5 + x_6$$



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x_i = gate's evaluated value

Objective: ??

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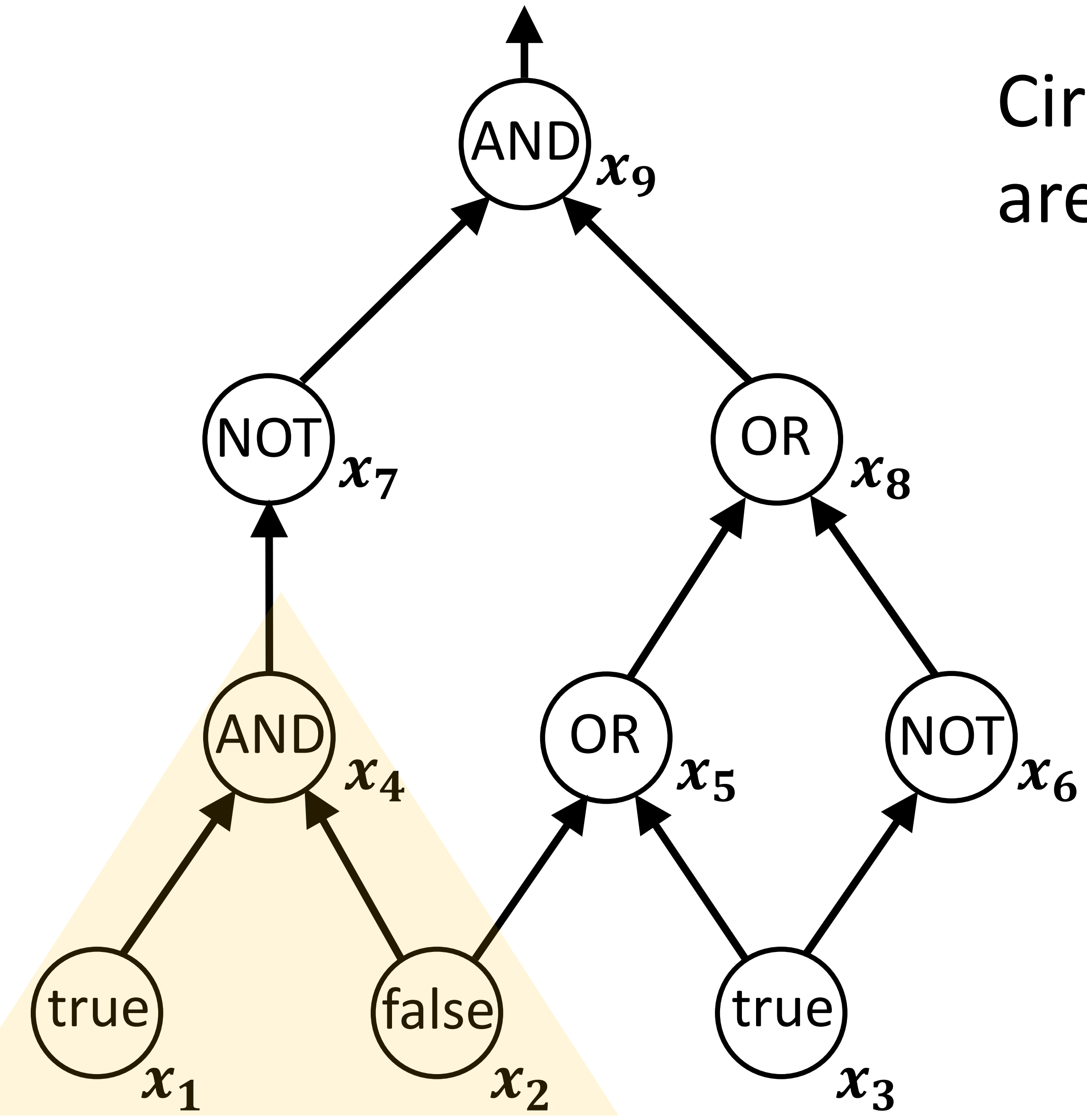
$x_7 = 1 - x_4$

$x_5 \geq x_2$
$x_5 \geq x_3$
$x_5 \leq x_2 + x_3$

$x_8 \geq x_5$
$x_8 \geq x_6$
$x_8 \leq x_5 + x_6$

??



Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value

Objective: ??

Subject to: $x_i \in \{0,1\}, \forall i$

$$x_1 = 1$$
$$x_2 = 0$$
$$x_3 = 1$$

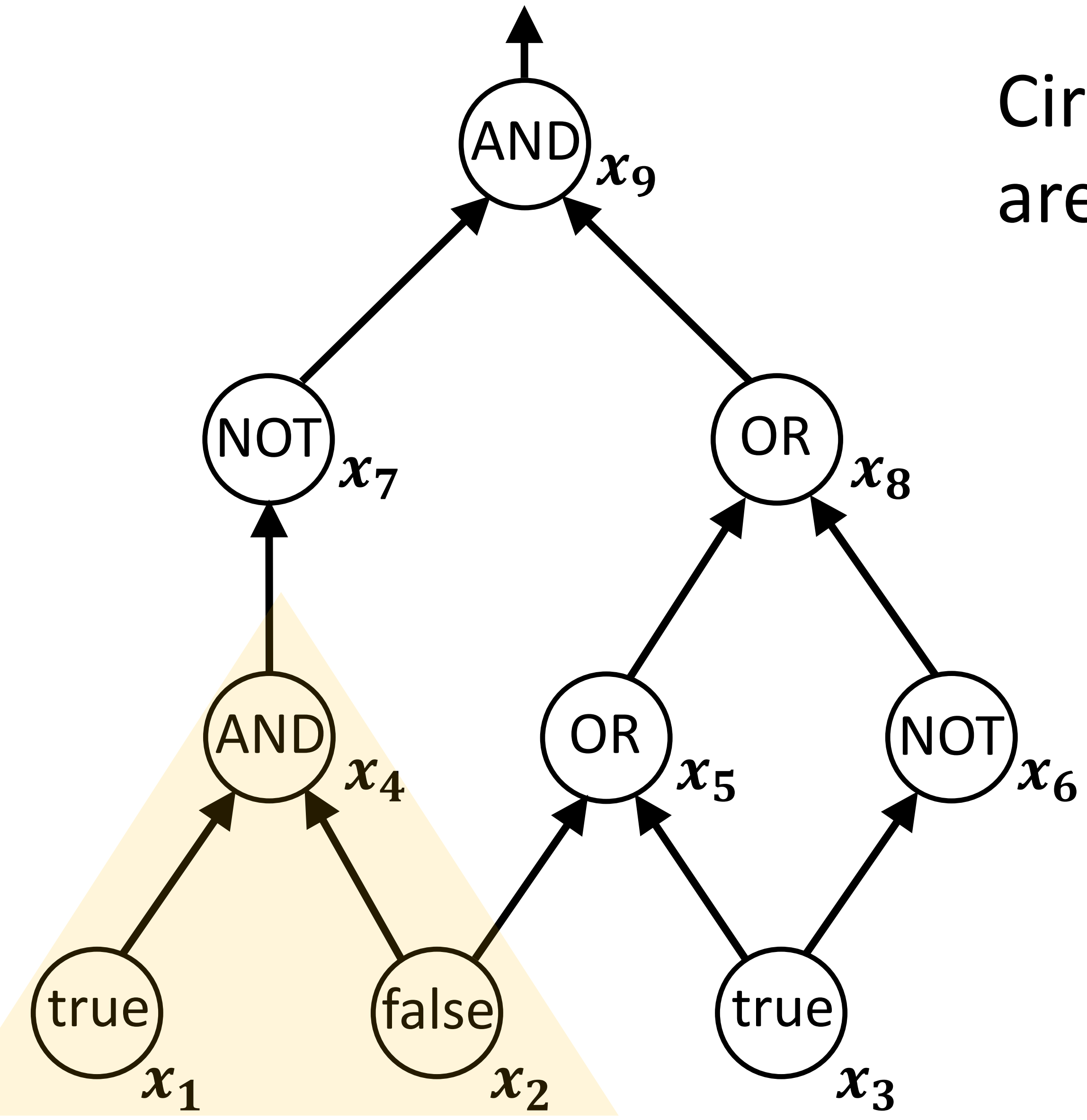
$$x_6 = 1 - x_3$$

$$x_7 = 1 - x_4$$

$$x_5 \geq x_2$$
$$x_5 \geq x_3$$
$$x_5 \leq x_2 + x_3$$

$$x_8 \geq x_5$$
$$x_8 \geq x_6$$
$$x_8 \leq x_5 + x_6$$

$$x_4 \leq x_1$$



Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value

Objective: ??

Subject to: $x_i \in \{0,1\}, \forall i$

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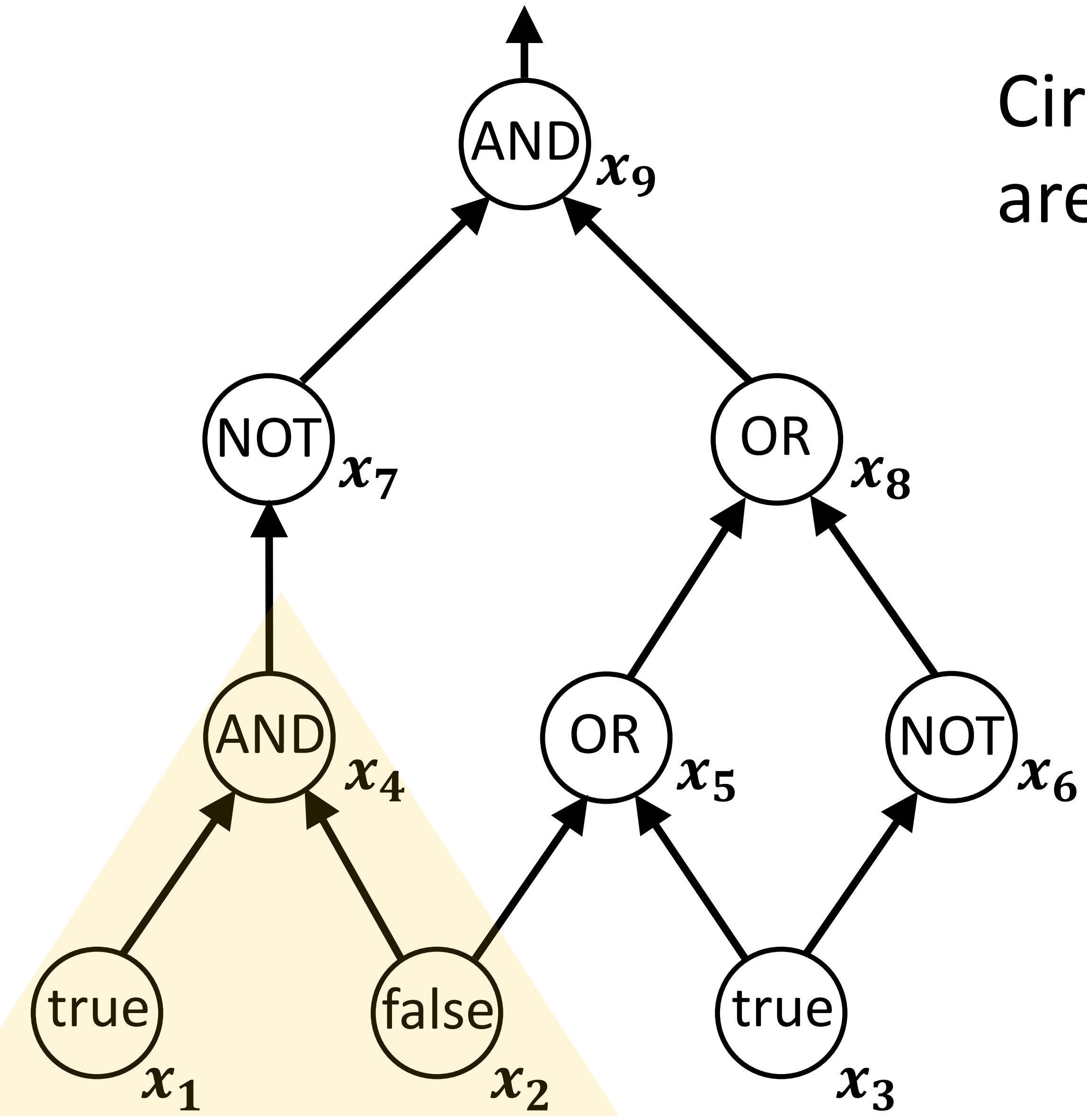
$$x_6 = 1 - x_3$$
$$x_7 = 1 - x_4$$
$$x_5 \geq x_2$$
$$x_5 \geq x_3$$
$$x_5 \leq x_2 + x_3$$

$$x_8 \geq x_5$$
$$x_8 \geq x_6$$
$$x_8 \leq x_5 + x_6$$

$$x_4 \leq x_1$$
$$x_4 \leq x_2$$



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x_i = gate's evaluated value

Objective: ??

Subject to: $x_i \in \{0,1\}, \forall i$

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$$x_2 = 0$$
$$x_3 = 1$$

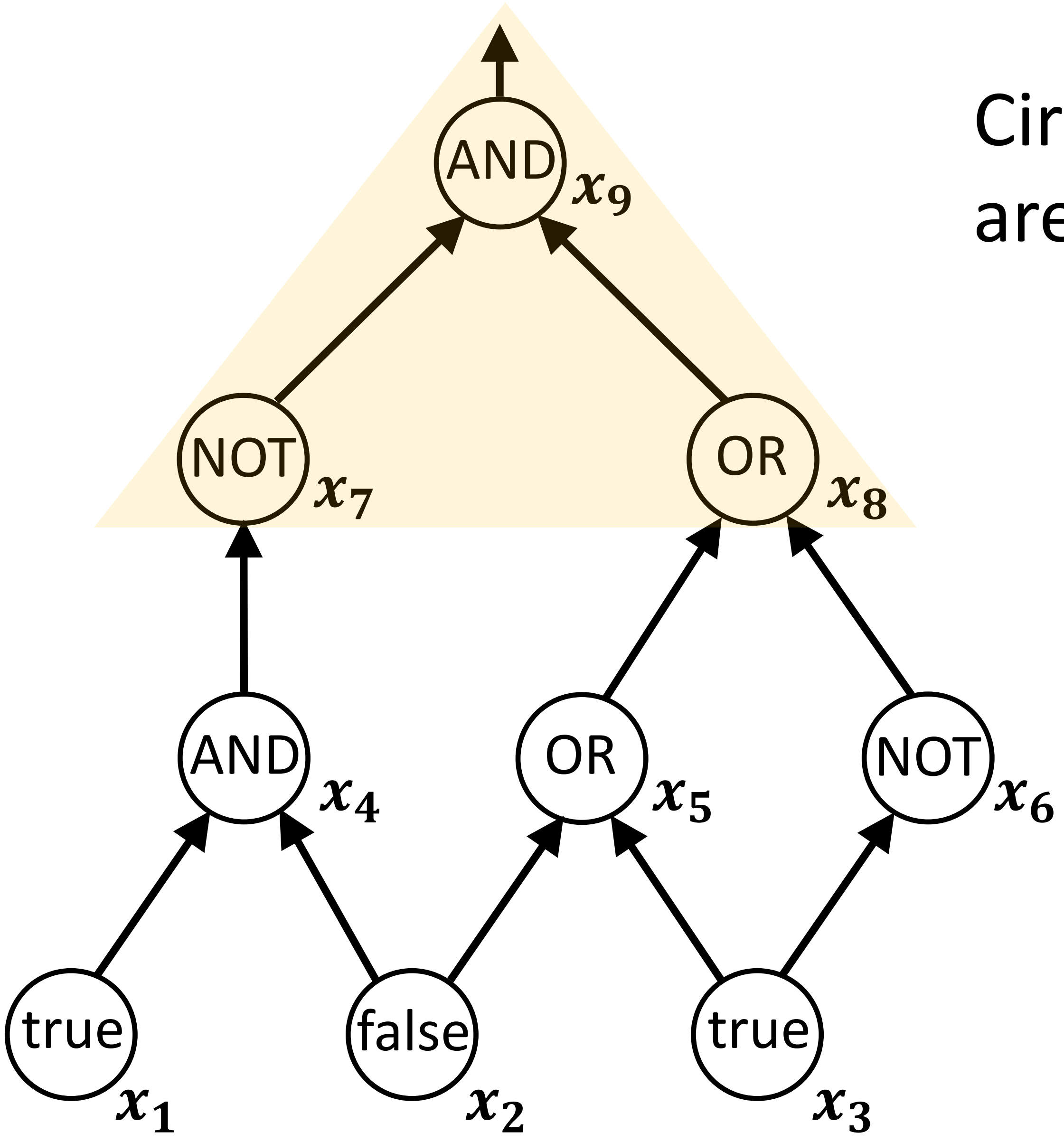
$$x_6 = 1 - x_3$$

$$x_7 = 1 - x_4$$

$$x_5 \geq x_2$$
$$x_5 \geq x_3$$
$$x_5 \leq x_2 + x_3$$

$$x_8 \geq x_5$$
$$x_8 \geq x_6$$
$$x_8 \leq x_5 + x_6$$

$$x_4 \leq x_1$$
$$x_4 \leq x_2$$
$$x_4 \geq x_1 + x_2 - 1$$



Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value

Objective: ??

Subject to: $x_i \in \{0,1\}, \forall i$

$$x_1 = 1$$
$$x_2 = 0$$
$$x_3 = 1$$

$$x_6 = 1 - x_3$$

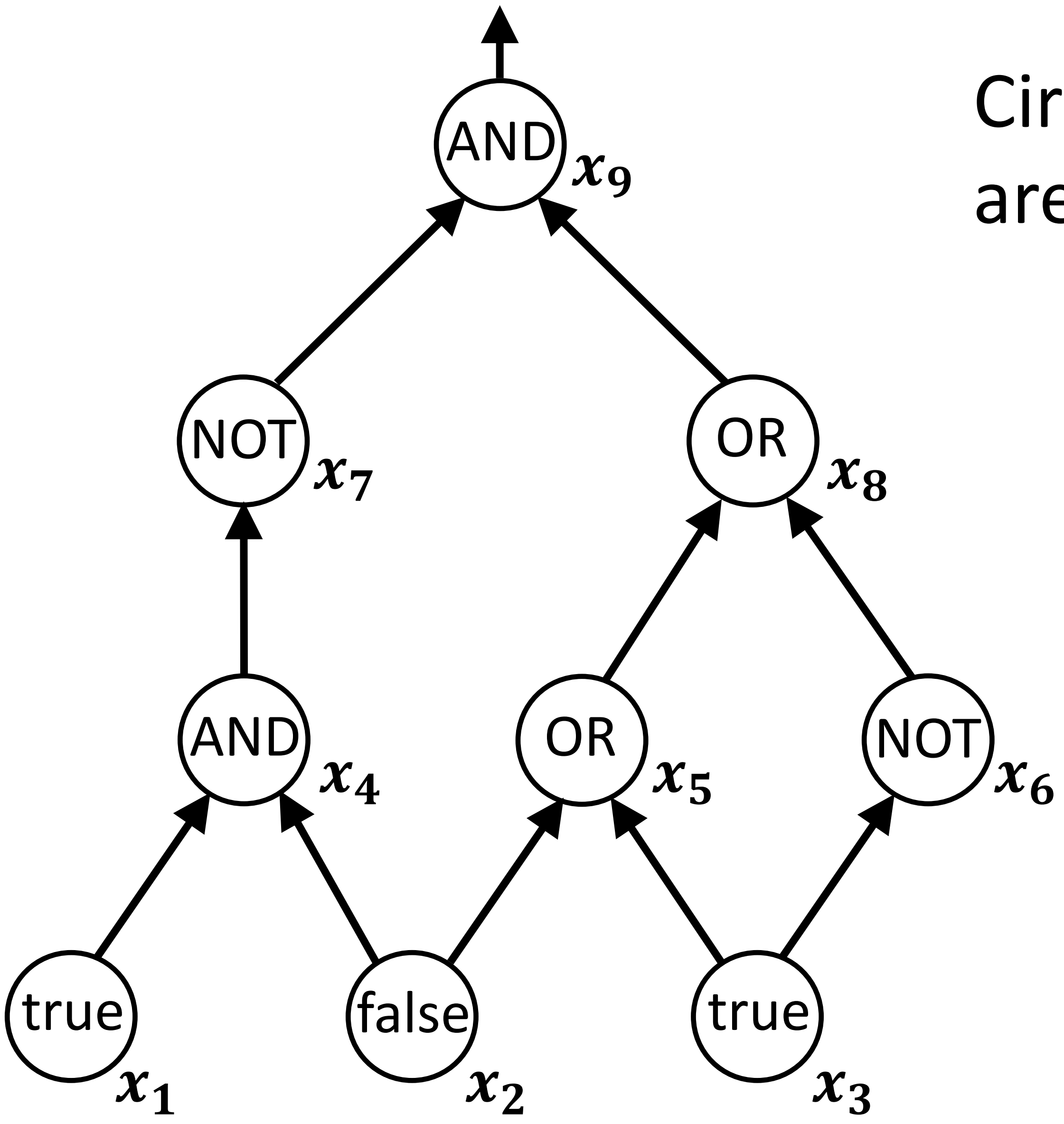
$$x_7 = 1 - x_4$$

$$x_5 \geq x_2$$
$$x_5 \geq x_3$$
$$x_5 \leq x_2 + x_3$$

$$x_8 \geq x_5$$
$$x_8 \geq x_6$$
$$x_8 \leq x_5 + x_6$$

$$x_4 \leq x_1$$
$$x_4 \leq x_2$$
$$x_4 \geq x_1 + x_2 - 1$$

$$x_9 \leq x_7$$
$$x_9 \leq x_8$$
$$x_9 \geq x_7 + x_8 - 1$$



Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value

Objective: ??

Subject to: $x_i \in \{0,1\}, \forall i$

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$$x_2 = 0$$
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$$x_6 = 1 - x_3$$
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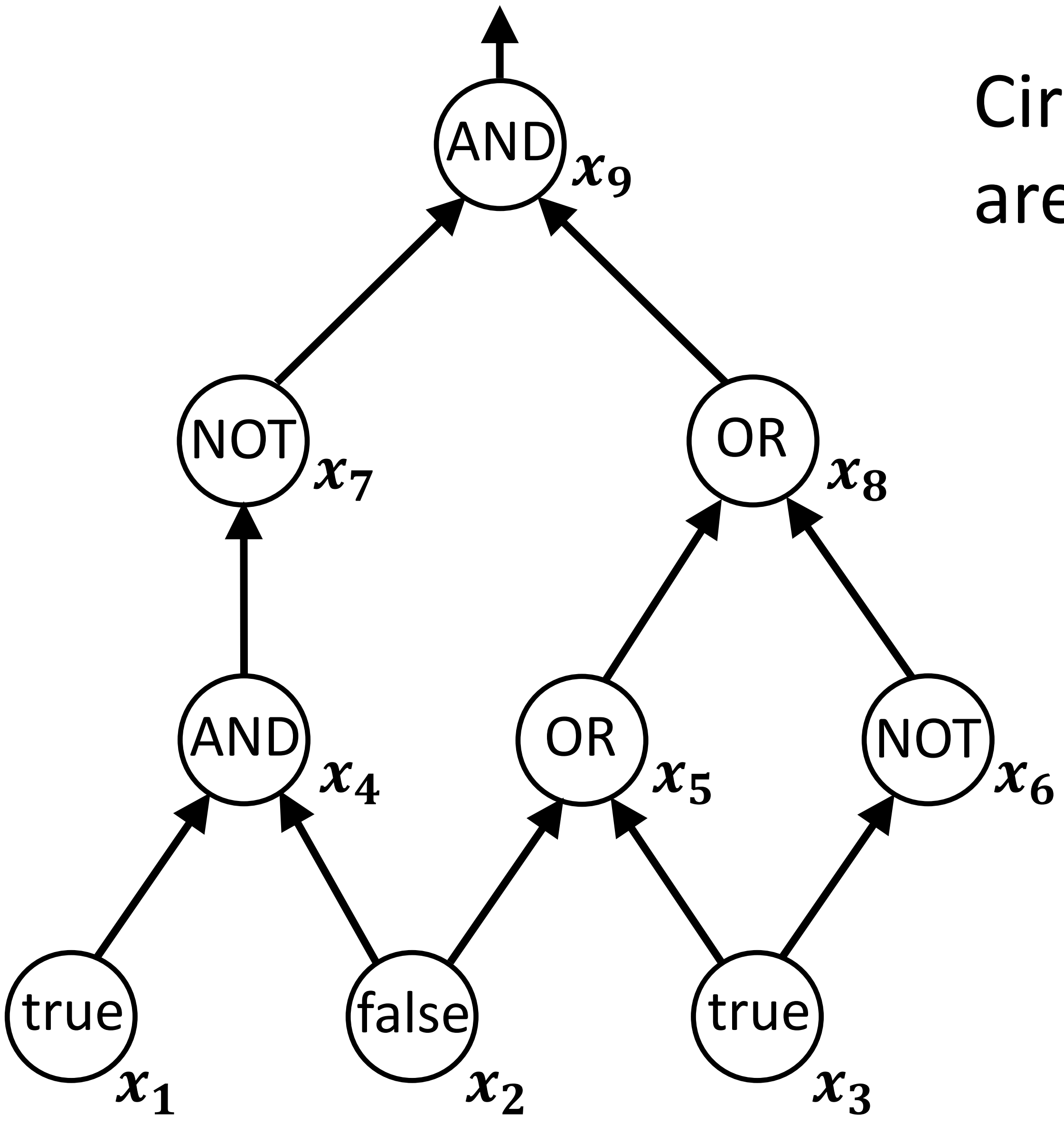
$$x_5 \geq x_3$$
$$x_5 \leq x_2 + x_3$$

$$x_8 \geq x_5$$
$$x_8 \geq x_6$$
$$x_8 \leq x_5 + x_6$$
$$x_4 \leq x_1$$
$$x_4 \leq x_2$$
$$x_4 \geq x_1 + x_2 - 1$$
$$x_9 \leq x_7$$
$$x_9 \leq x_8$$
$$x_9 \geq x_7 + x_8 - 1$$



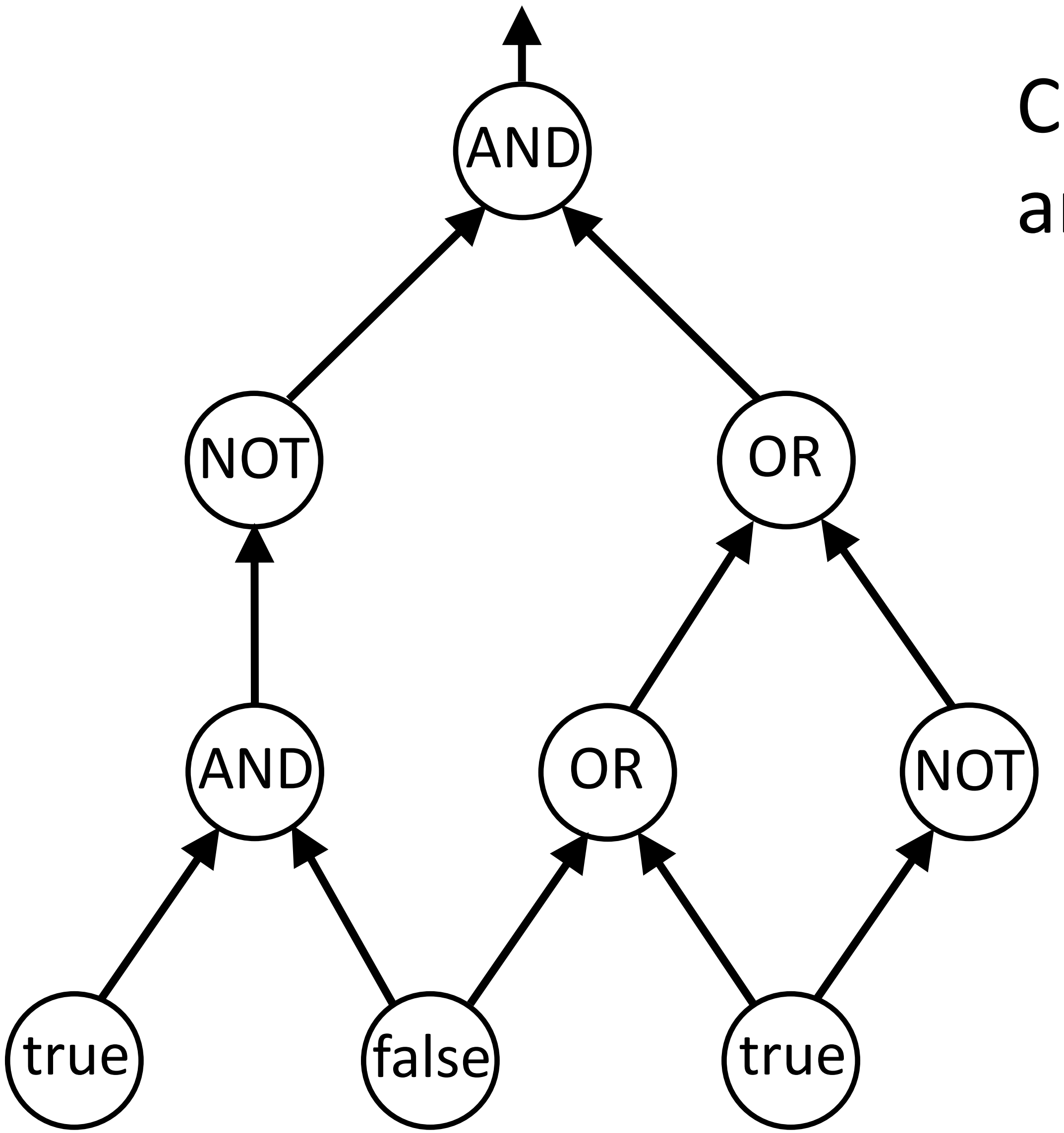
Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value

Objective: $\min 0$

Subject to: $x_i \in \{0,1\}, \forall i$

$$x_1 = 1$$
$$x_2 = 0$$
$$x_3 = 1$$
$$x_6 = 1 - x_3$$
$$x_7 = 1 - x_4$$
$$x_5 \geq x_2$$
$$x_5 \geq x_3$$
$$x_5 \leq x_2 + x_3$$
$$x_8 \geq x_5$$
$$x_8 \geq x_6$$
$$x_8 \leq x_5 + x_6$$
$$x_4 \leq x_1$$
$$x_4 \leq x_2$$
$$x_4 \geq x_1 + x_2 - 1$$
$$x_9 \leq x_7$$
$$x_9 \leq x_8$$
$$x_9 \geq x_7 + x_8 - 1$$



Circuit Value Problem: When the gates are evaluated, is the output true?

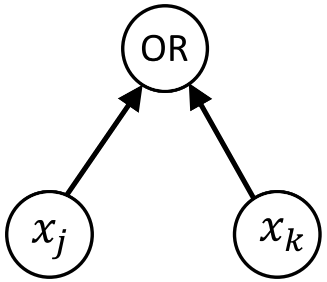
$x_i =$ gate's evaluated value

Objective: $\min 0$

Subject to: $x_i \in \{0,1\}$

$x_i = 1$

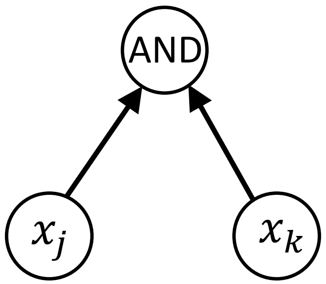
$x_i = 0$



$x_i \geq x_j$

$x_i \geq x_k$

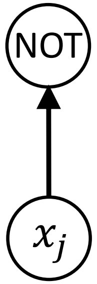
$x_i \leq x_j + x_k$



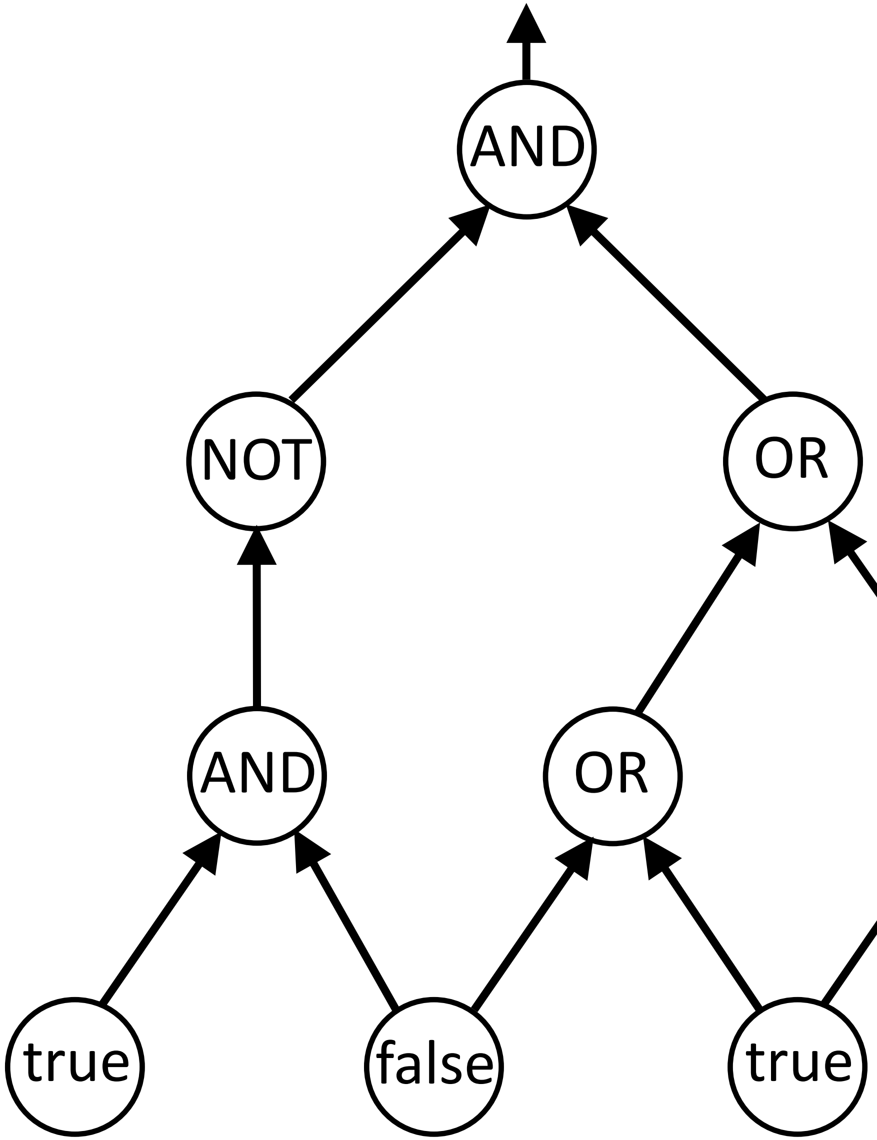
$x_i \leq x_j$

$x_i \leq x_k$

$x_i \geq x_j + x_k - 1$



$x_i = 1 - x_j$



Will answer be an integer?

Circuit Value Problem: When the gates are evaluated, is the output true?

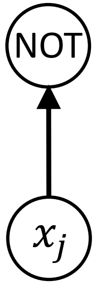
x_i = gate's evaluated value

Objective: $\min 0$

Subject to: $x_i \in [0,1]$

$x_i = 1$

$x_i = 0$



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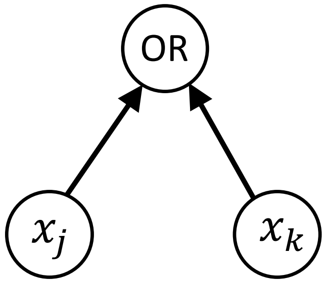

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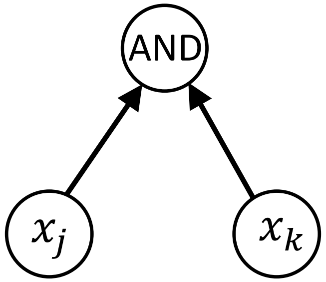

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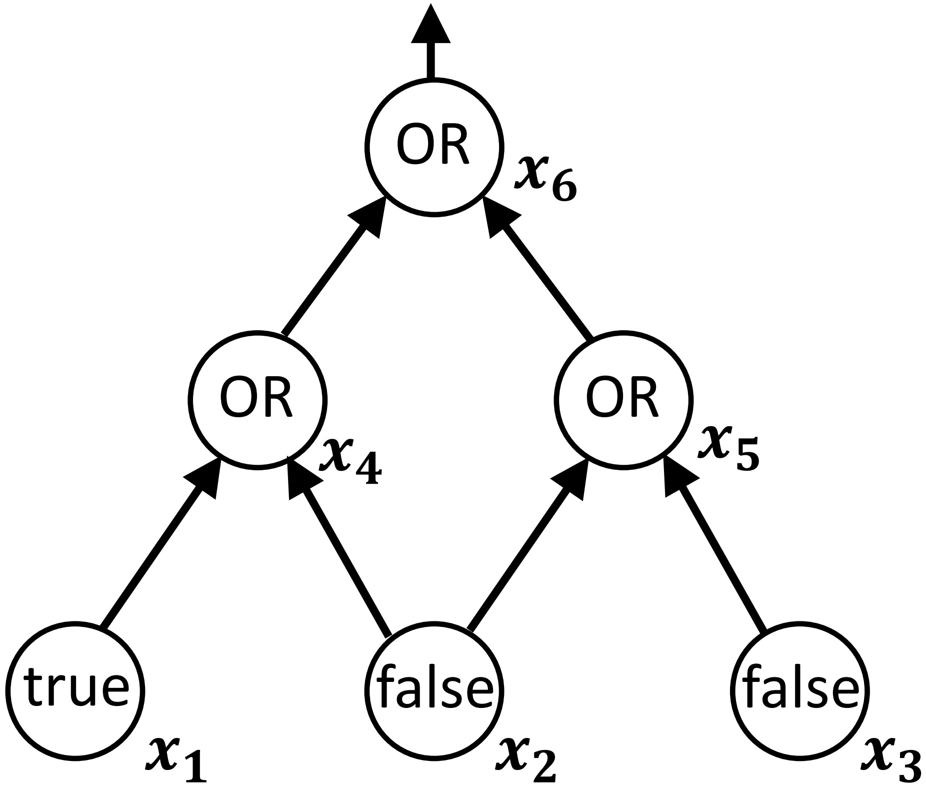

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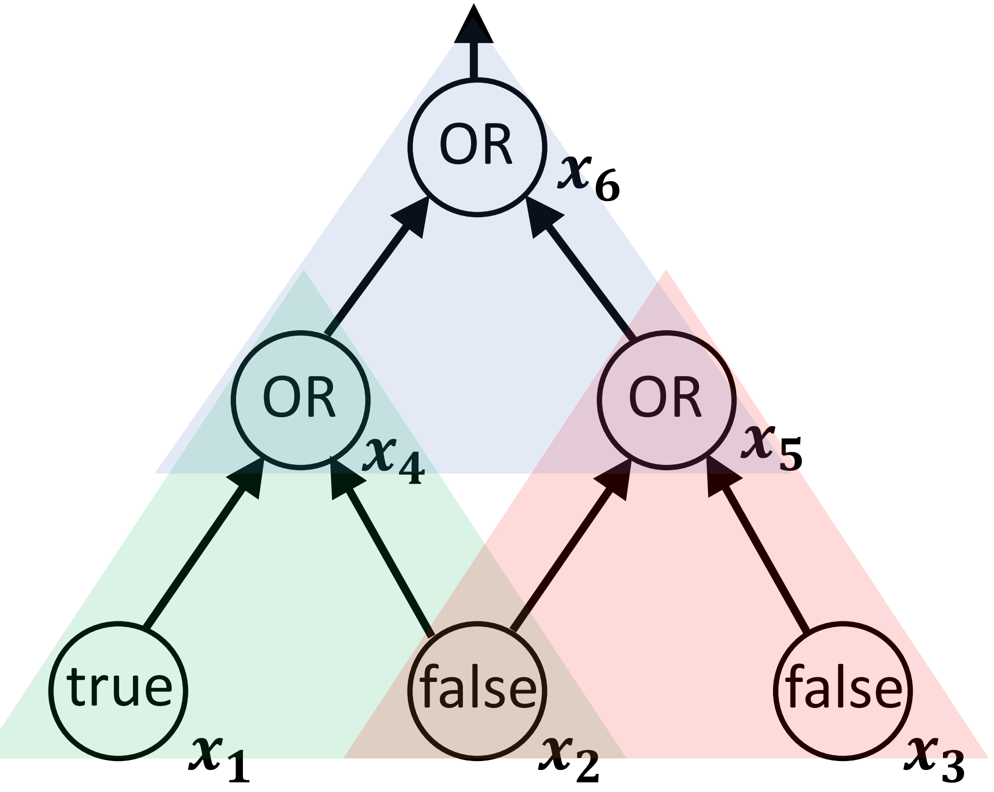
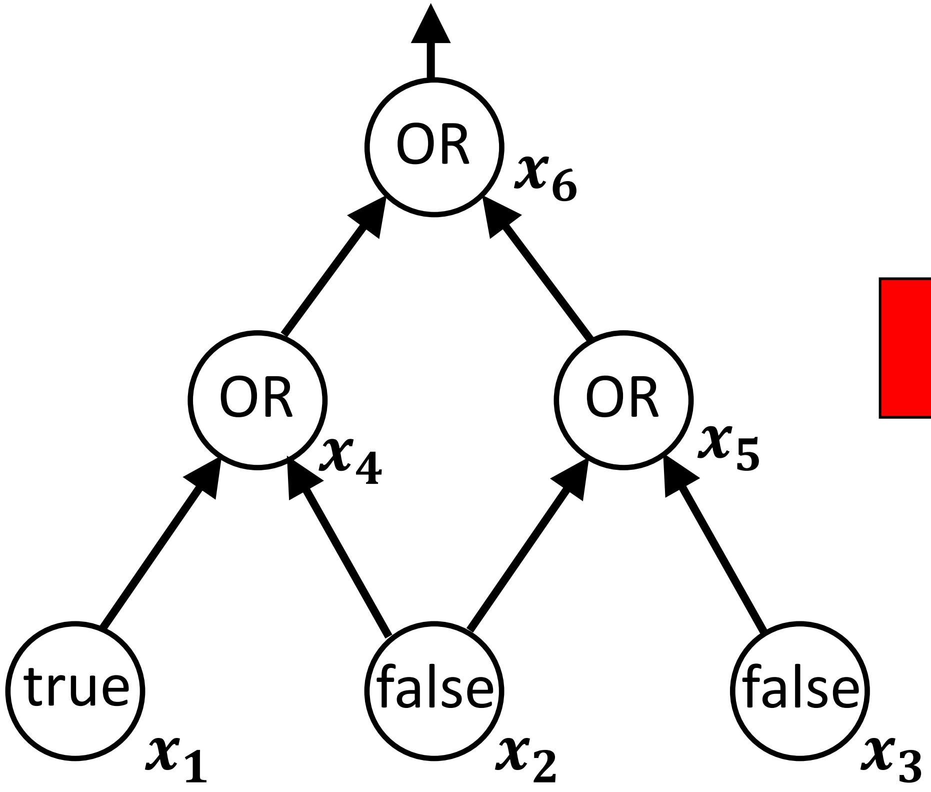


$$
A = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \end{array}
\begin{array}{c}
\begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\
\begin{pmatrix}
1 & 0 & 0 & -1 & 0 & 0 \\
0 & 1 & 0 & -1 & 0 & 0 \\
-1 & -1 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & -1 & 0 \\
0 & 0 & 1 & 0 & -1 & 0 \\
0 & -1 & -1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & -1 \\
0 & 0 & 0 & 0 & 1 & -1 \\
0 & 0 & 0 & -1 & -1 & 1
\end{pmatrix}
\end{array}
$$

Will answer be an integer?



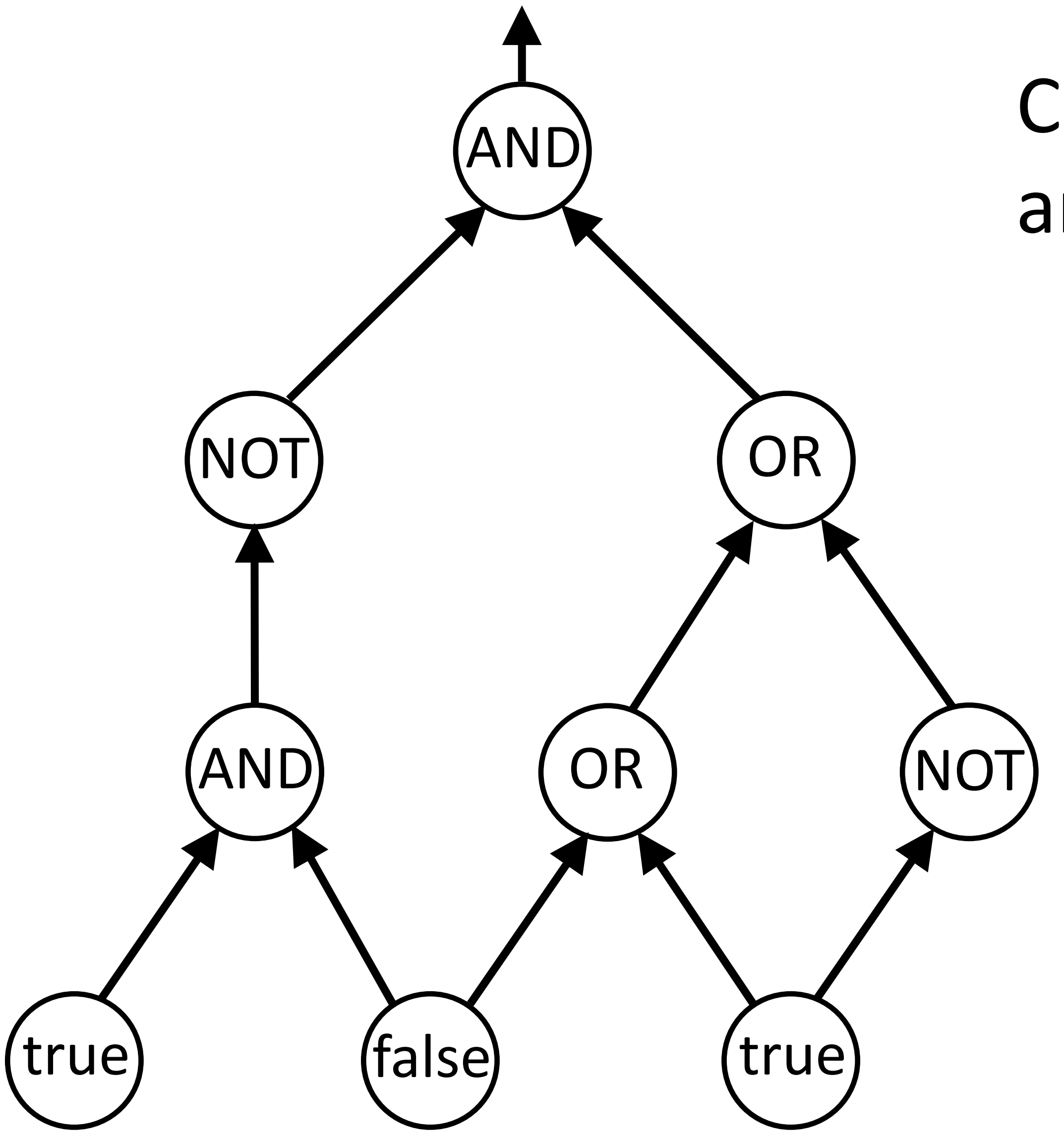
$$A = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \end{array} \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ -1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & -1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 & -1 & 1 \end{pmatrix} \end{array}$$

$$\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & -1 \end{bmatrix}$$

det = -2

Will answer be an integer?

Circuit Value Problem: When the gates are evaluated, is the output true?



Will answer be an integer?

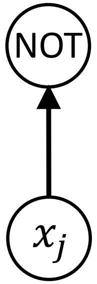
x_i = gate's evaluated value

Objective: $\min 0$

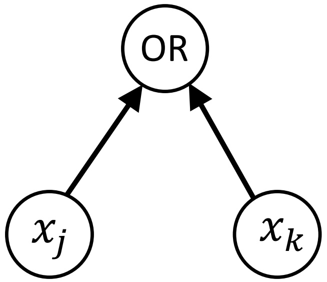
Subject to: $x_i \in [0,1]$

$x_i = 1$

$x_i = 0$



$x_i = 1 - x_j$



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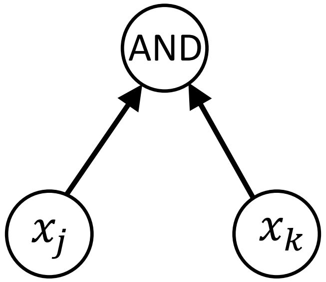

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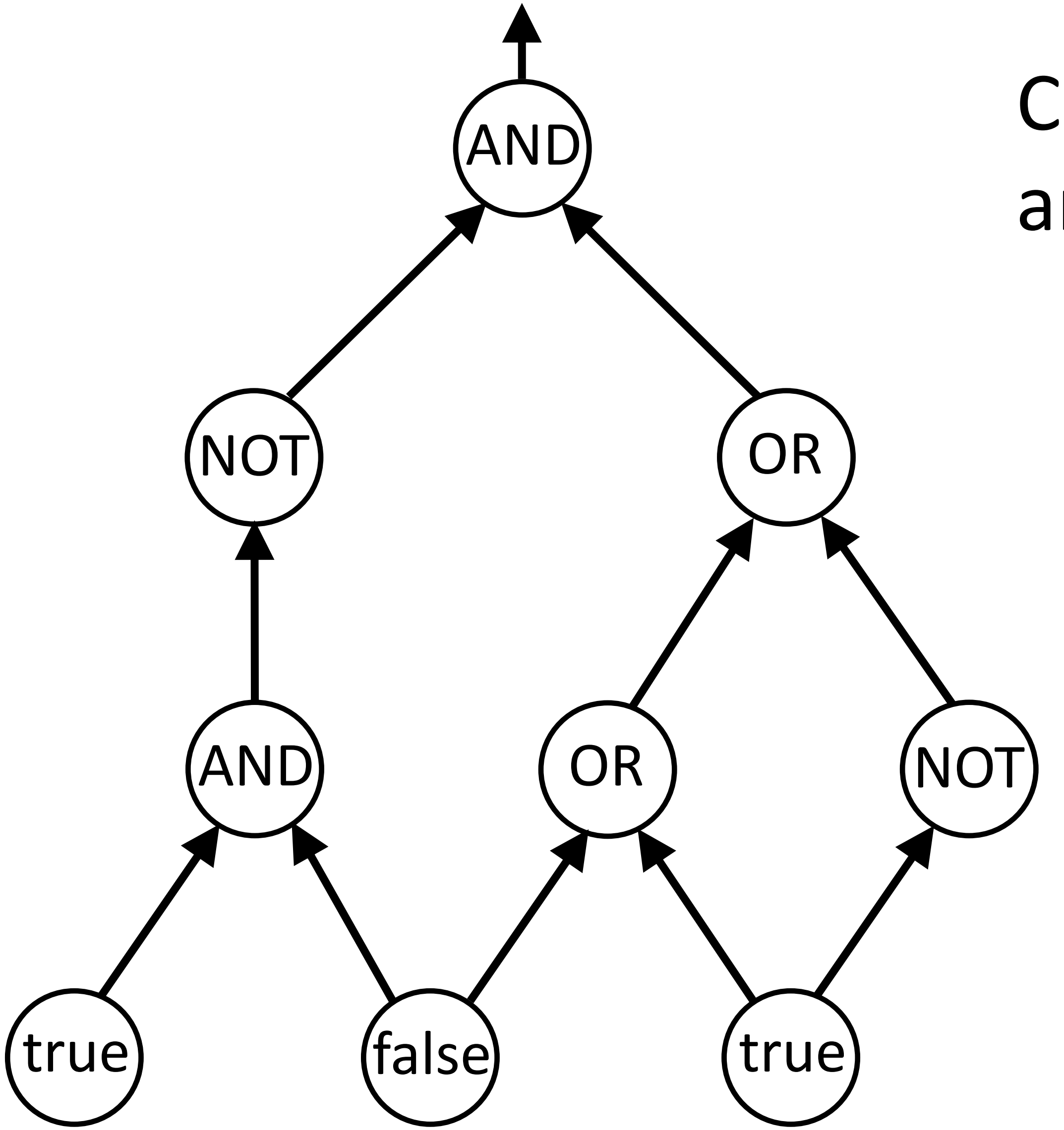

Will answer be an integer?

Yes! Input are integers and constraints keep them integers.

Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value

Objective: $\min 0$

Subject to: $x_i \in [0,1]$

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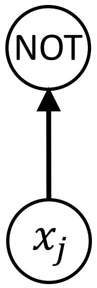

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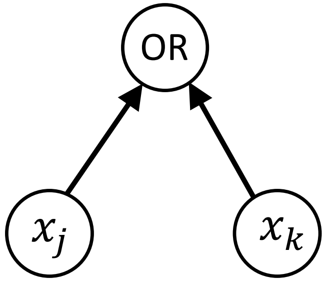

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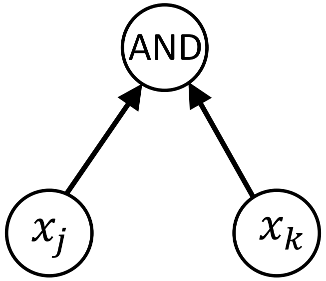

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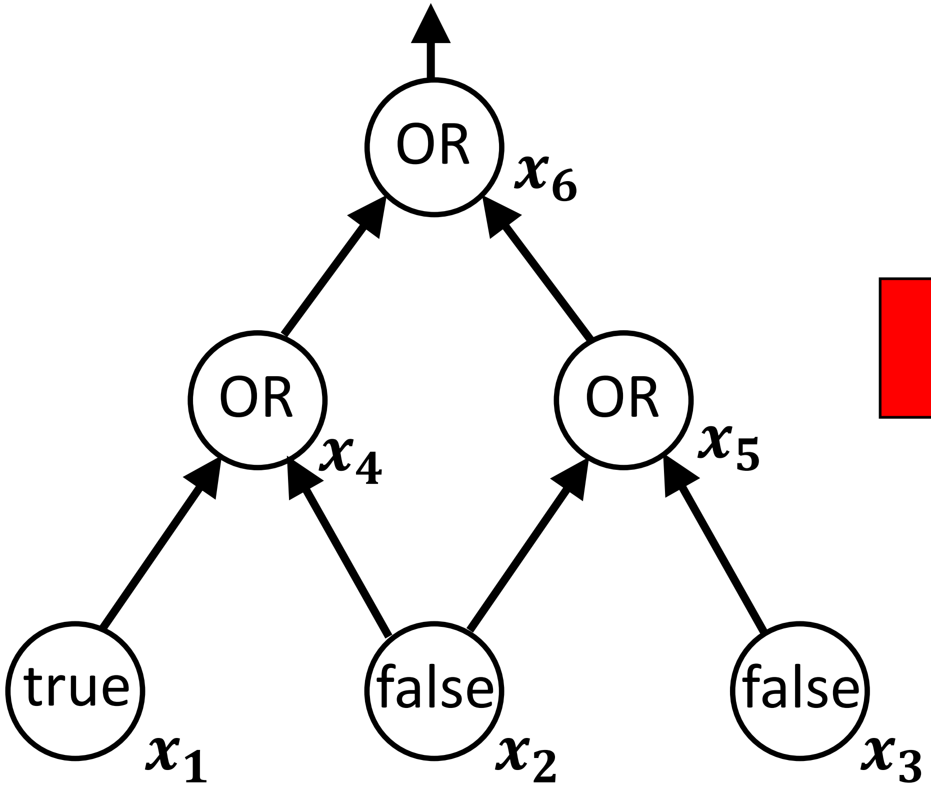

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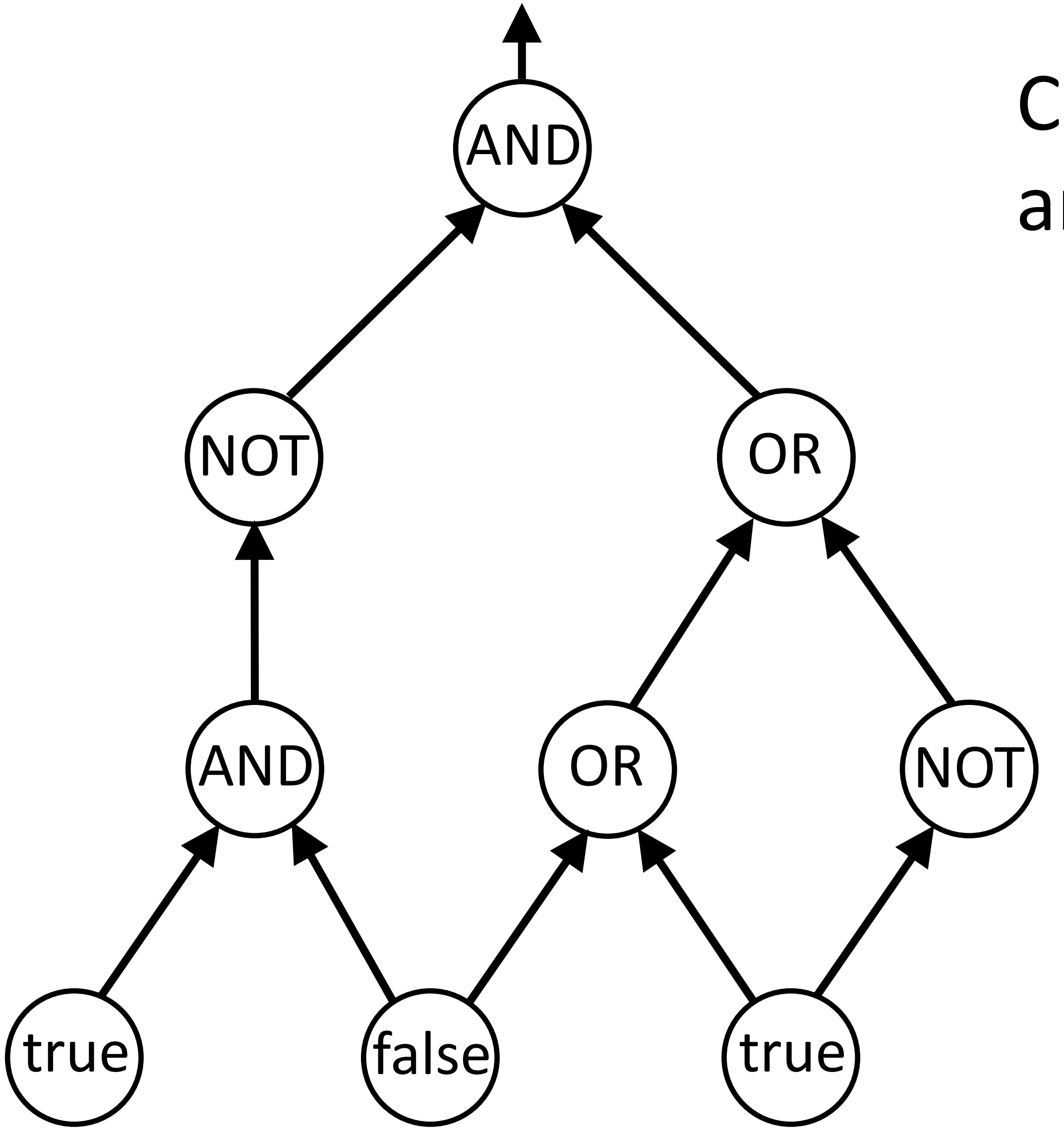
$$\begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & -1 \end{pmatrix} \quad \text{det} = -2$$

Will answer be an integer?

Total unimodularity is not the only path to integer solutions.

Circuit Value Problem: When the gates are evaluated, is the output true?



Anything that can run on a computer in poly time can be solved with an LP in poly time!

x_i = gate's evaluated value

Objective: $\min 0$

Subject to: $x_i \in [0,1]$

$x_i = 1$

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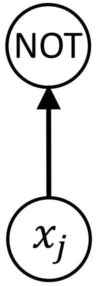

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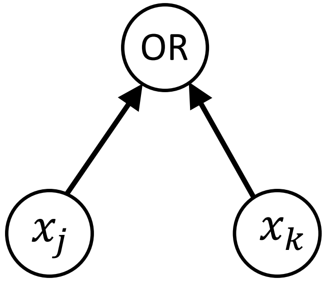

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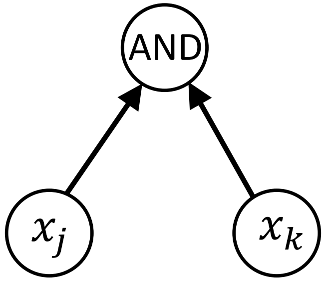

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NP

$$P \left\{ \begin{array}{l} \boldsymbol{P} \text{ is the set of problems that are} \\ \text{solvable by an algorithm whose} \\ \text{running time is polynomial time.} \end{array} \right.$$

NP

P — $\boldsymbol{P}$ is the set of problems that are solvable by an algorithm whose running time is polynomial time.

NP — Set of problems that are verifiable in polynomial time.

$SUBSET - SUM$

Claim: $SUBSET - SUM = \{\langle S, t\rangle : S = \{x_1, \ldots, x_n\}$, and there exists some $\{y_1, \ldots, y_m\} \subseteq S$ such that $\sum y_i = t\} \in NP$.

Example:

$\langle\{4, 11, 16, 21, 27\}, 25\rangle \in SUBSET - SUM$ since $4 + 21 = 25$

$\langle\{1, 2, 3\}, 10\rangle \notin SUBSET - SUM$, since no subsets sum to 10.

$SUBSET - SUM$

Claim: $SUBSET - SUM = \{\langle S, t\rangle : S = \{x_1, \ldots, x_n\}$, and there exists some $\{y_1, \ldots, y_m\} \subseteq S$ such that $\sum y_i = t\} \in NP$.

Proof:

Solver: Is $\langle S, t\rangle \in SUBSET - SUM$?

Verifier: Is $\langle S, t\rangle \in SUBSET - SUM$, given a <u>candidate solution</u>?

Set of numbers.

$SUBSET - SUM$

Claim: $SUBSET - SUM = \{\langle S, t\rangle : S = \{x_1, \dots, x_n\}$, and there exists some $\{y_1, \dots, y_m\} \subseteq S \text{ such that } \sum y_i = t\} \in NP$.

Proof: Build a polynomial time verifier.

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Proof: Build a polynomial time verifier.

On input $\langle\langle S, t\rangle, C\rangle$, where C is a collection of numbers

1. Test if every element from C is in S.
2. Test if the sum of C is t.
3. If both pass, <u>accept</u>. Otherwise, <u>reject</u>.

$SUBSET - SUM$

Claim: $SUBSET - SUM = \{\langle S, t\rangle : S = \{x_1, \dots, x_n\}$, and there exists some $\{y_1, \dots, y_m\} \subseteq S$ such that $\sum y_i = t\} \in NP$.

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Verified in $O(n)$ time, therefore $SUBSET - SUM \in NP$.

P vs NP

P — $\boldsymbol{P}$ is the set of problems that are solvable by an algorithm whose running time is polynomial time.

NP — Set of problems that are verifiable in polynomial time.

P vs NP

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How does P relate to NP?

P vs NP

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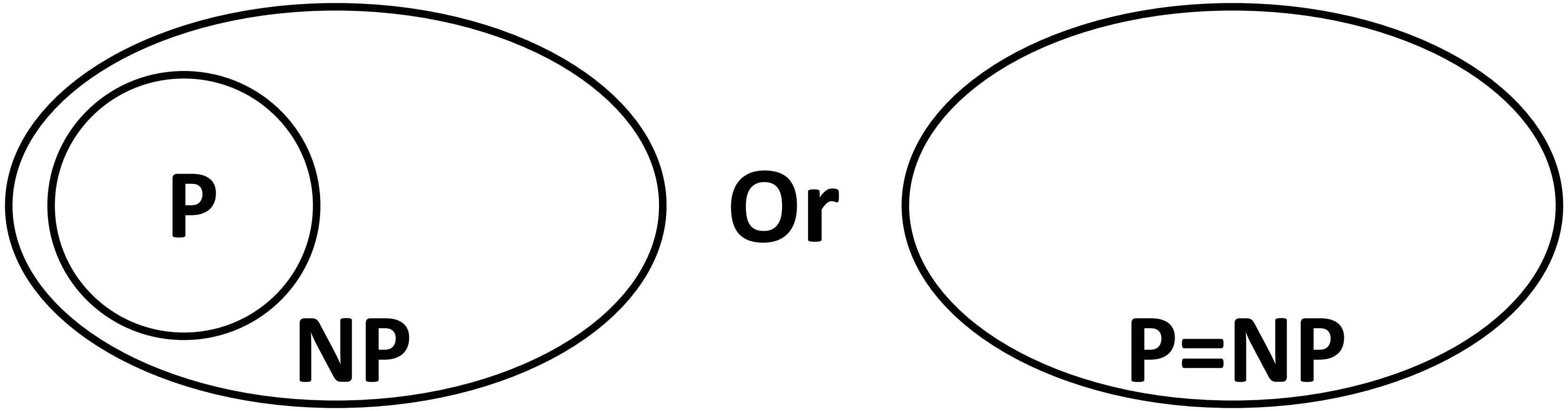
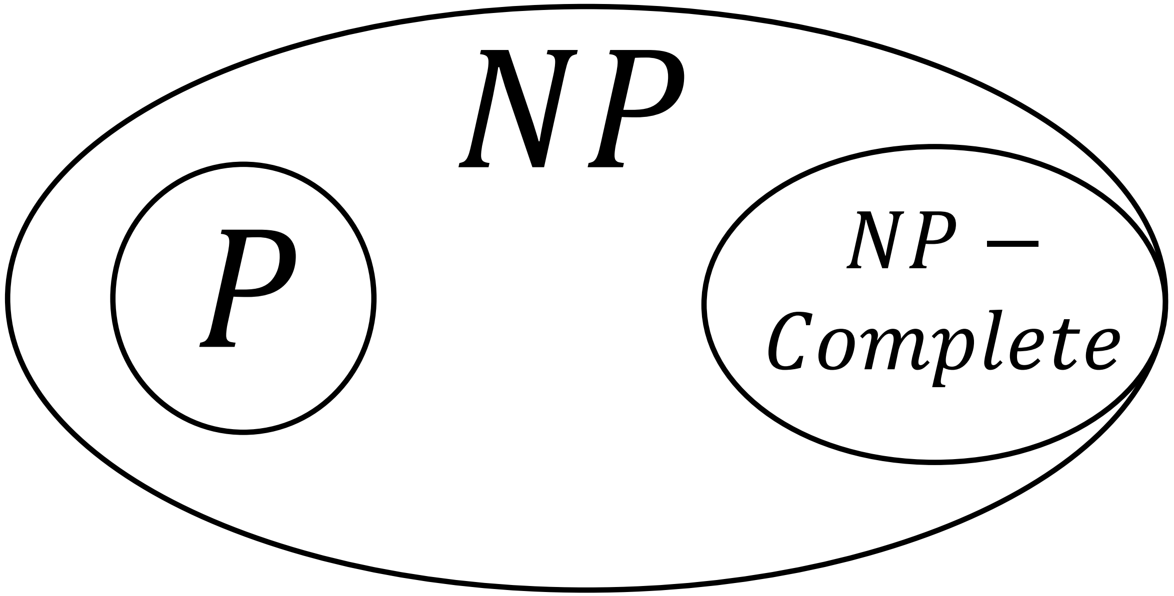
How does P relate to NP?



NP
$Complete$

Set of problems in NP whose algorithms can solve any other problem in NP with polynomial extra time.



$$NP\ Complete \left\{ \begin{array}{l} \text{Set of problems in NP whose algorithms} \\ \text{can solve any other problem in NP with} \\ \text{polynomial extra time.} \end{array} \right.$$

Problem A: An $NP - Complete$ problem.

Problem B: Some random problem in NP.

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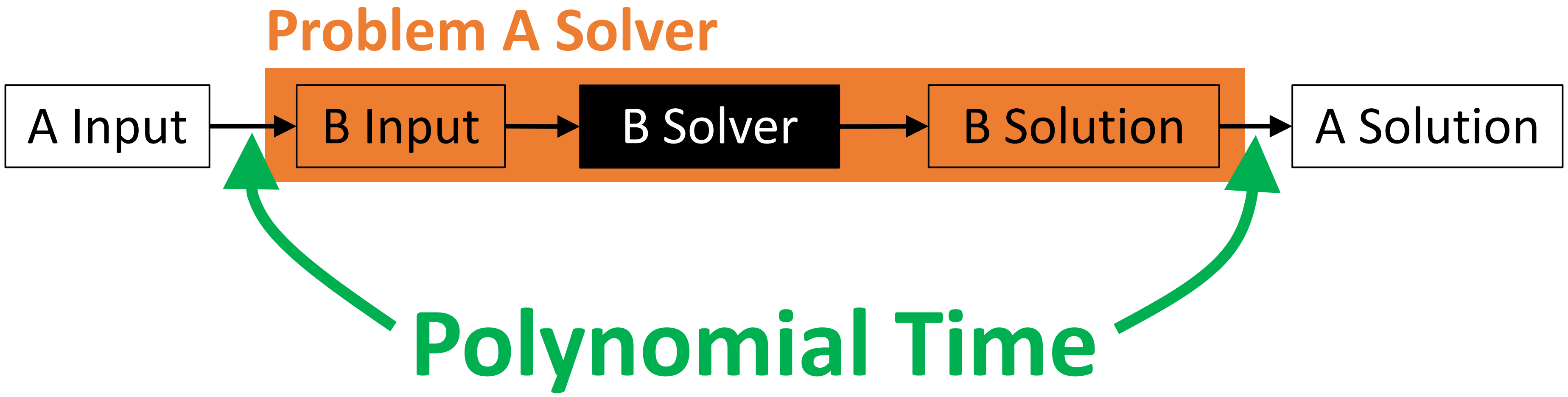

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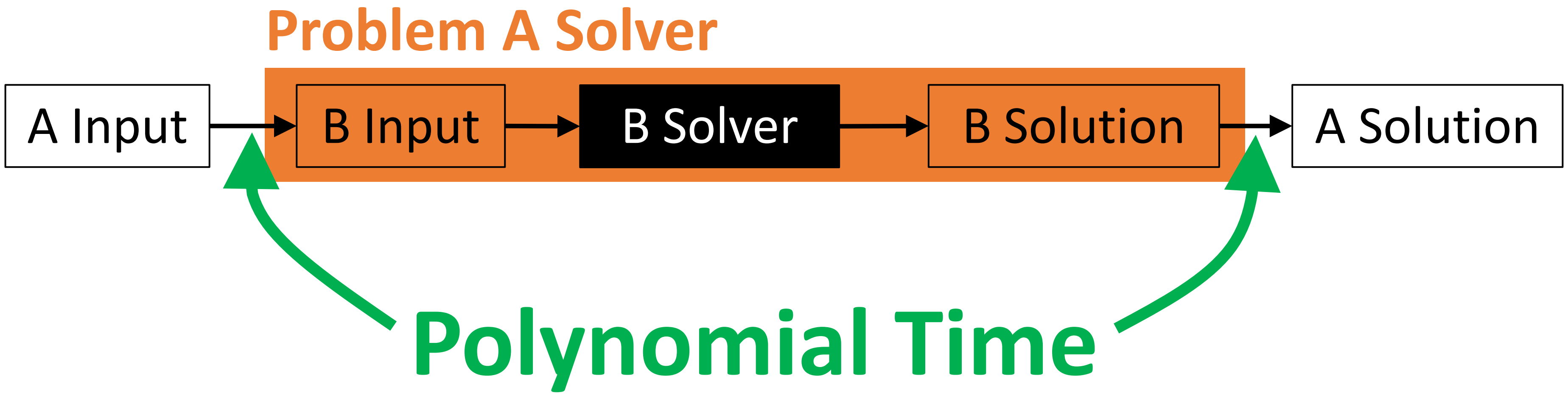


$$NP\ Complete \left\{ \text{Set of problems in NP whose algorithms can solve any other problem in NP with polynomial extra time.} \right.$$

Problem A: An $NP - Complete$ problem.

Problem B: ~~Some random problem in~~ $\cancel{NP}$~~.~~ An $NP - Complete$ problem.

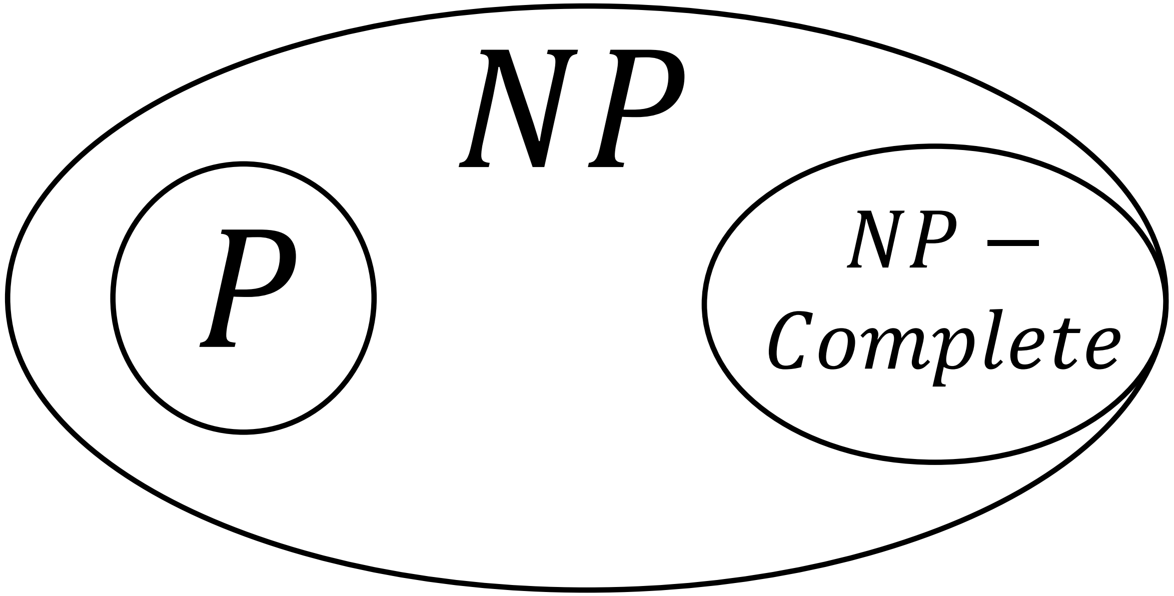
Consequences:

- A polynomial time algorithm for ***any*** $NP - Complete$ problem gives a polynomial time algorithm for ***every*** problem in NP.
- The thing that (possibly) makes one NP-C problem unsolvable in polynomial time is the exact same thing that makes every other NP-C problem unsolvable in polynomial time.

Polynomial Time Reductions

A <u>reduces to</u> B if A can be solved with a solver for B.

$NP - Complete$



B is in $NP - Complete$ if it satisfies two conditions:

1. $B \in NP$.
2. For every $A \in NP$, $A \leq_P B$.

B is in $NP - Complete$ if it satisfies two conditions:

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$NP - Complete$

B is in $NP - Complete$ if it satisfies two conditions:

1. $B \in NP$.
2. For some $NP - C$ problem A, $A \leq_P B$.

Cook-Levin Theorem:

The Boolean Satisfiability problem (SAT) is in $NP - C$.

$$\phi = (x_1 \vee x_1 \vee x_2) \wedge (\overline{x_1} \vee \overline{x_2} \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_2 \vee x_2)$$

$$(F \vee F \vee T) \qquad (T \vee F \vee F) \qquad (T \vee T \vee T)$$

$$T \qquad T \qquad T$$

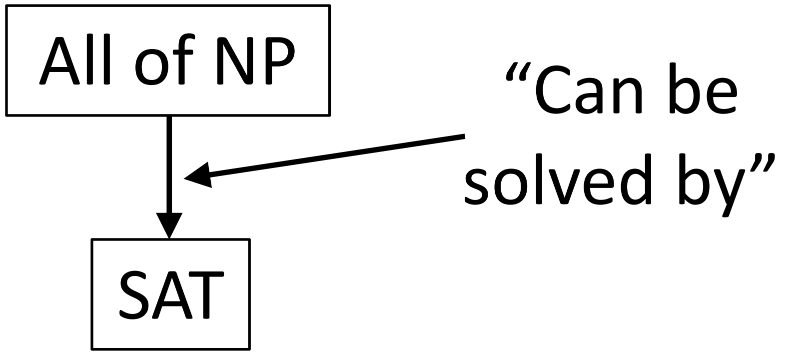
$$T$$

Can you set the variables to **true** or **false** so that ϕ evaluates to **true**?

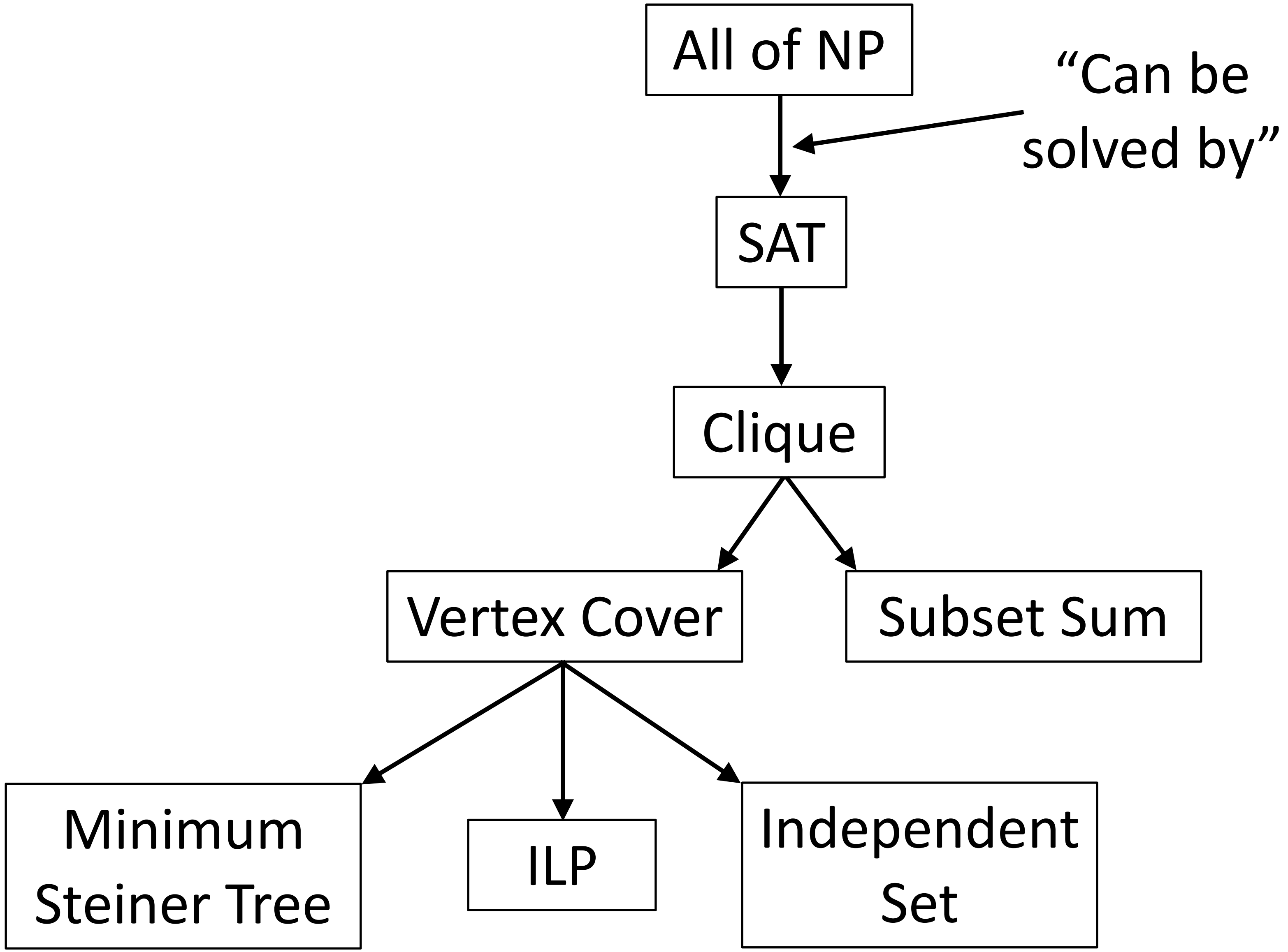
$x_1 = false$

$x_2 = true$

Cook-Levin Theorem



$NP - C$


All of NP
"Can be solved by"
SAT
Clique
Vertex Cover
Subset Sum
Minimum Steiner Tree
ILP
Independent Set