Introduction
CSCI 532

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How can we solve this problem?

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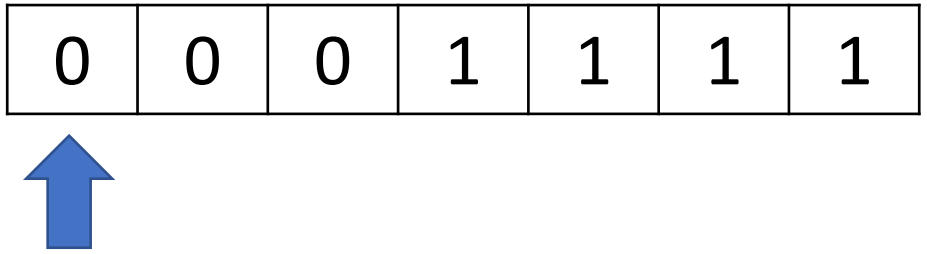

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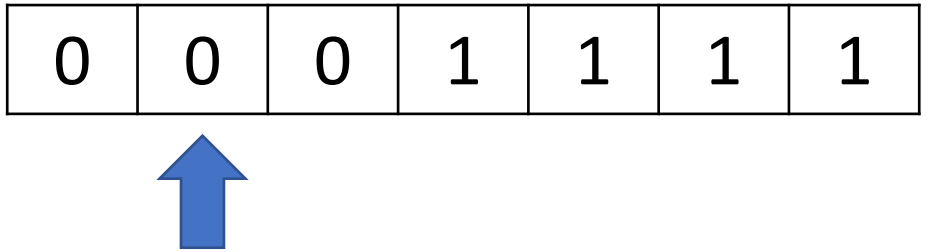

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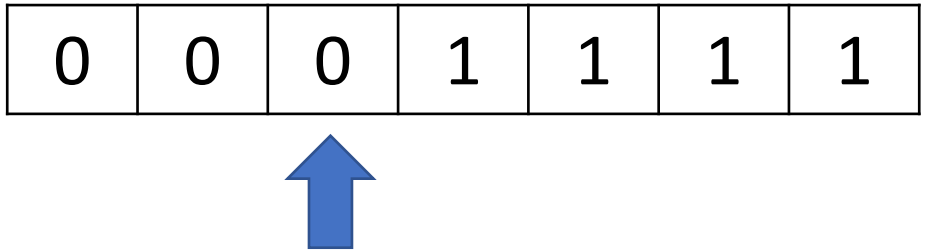

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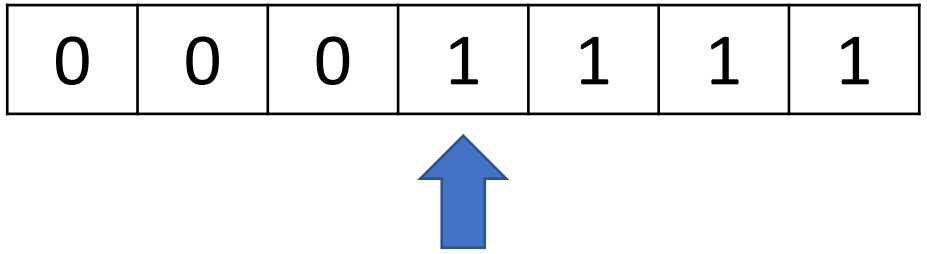

count = 1

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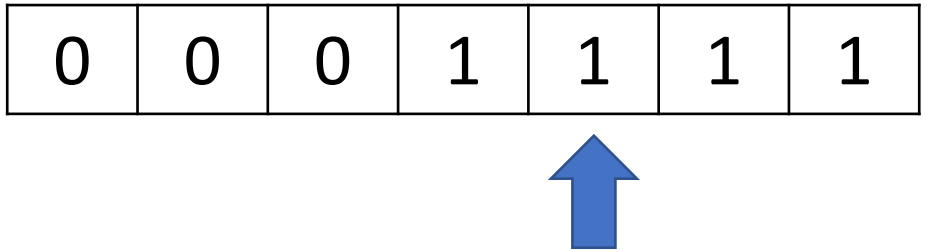

count = 2

Problem: Given a sorted array of 0's followed by 1's (e.g., [0,0,0,1,1,1,1]), return the number of 1's in the array.

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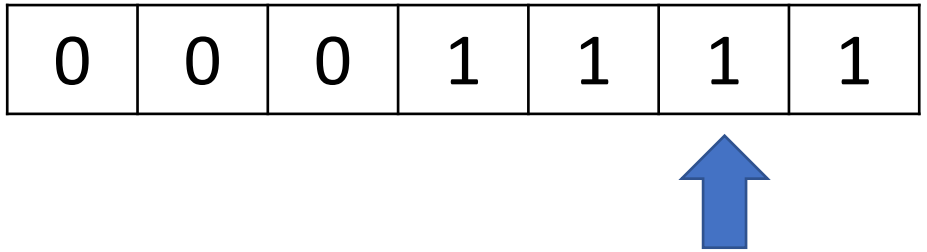

count = 3

Problem: Given a sorted array of 0's followed by 1's (e.g., [0,0,0,1,1,1,1]), return the number of 1's in the array.

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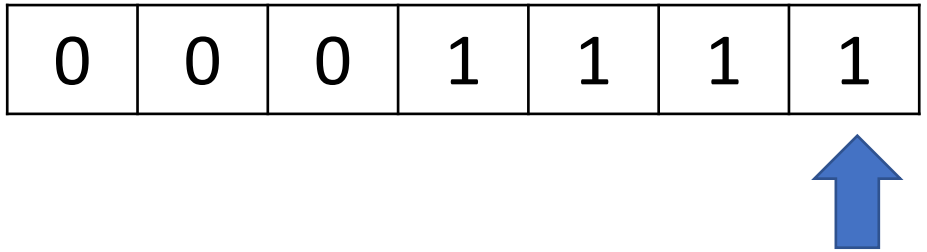

count = 4

Problem: Given a sorted array of 0's followed by 1's (e.g., [0,0,0,1,1,1,1]), return the number of 1's in the array.

```
countOnes_1(array a):
   count = 0
   for x in a:
      if x == 1:
         count += 1
   return count
```

```
countOnes_2(array a):
   left = 0, right = a.length - 1
   first_one = -1
   while left <= right:
      mid = (left + right) / 2
      if a[mid] == 1:
         first_one = mid
         right = mid - 1
      else:
         left = mid + 1
   if first_one == -1:
      return 0
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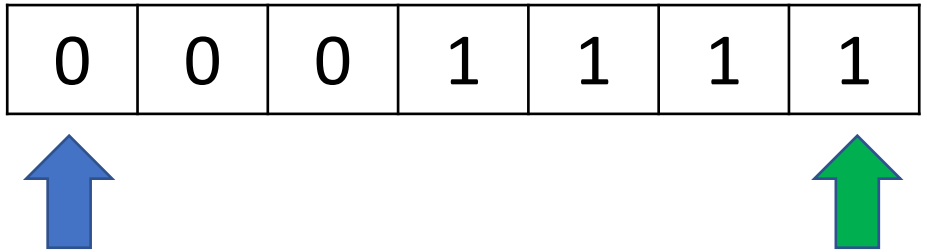


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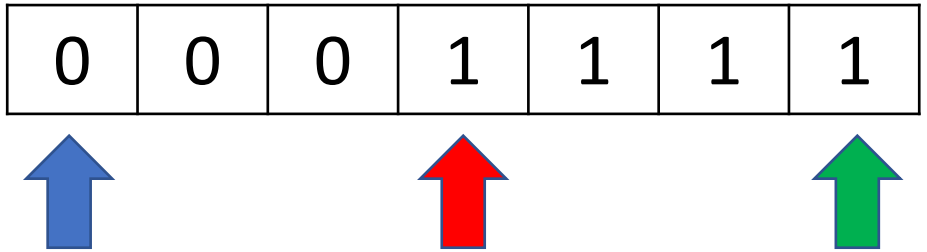


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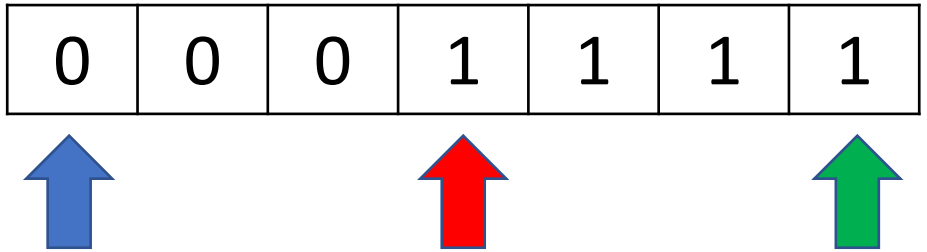

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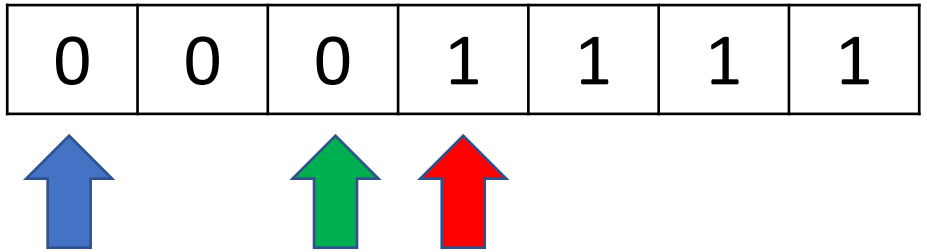

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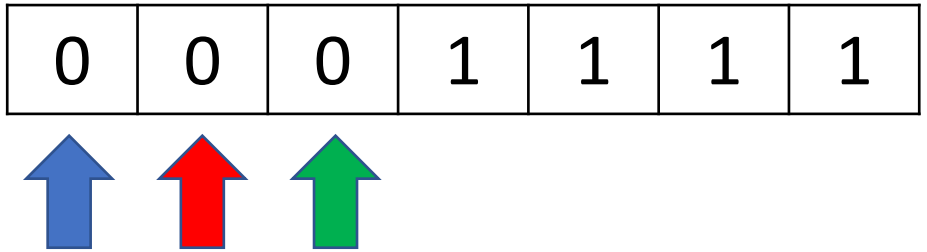

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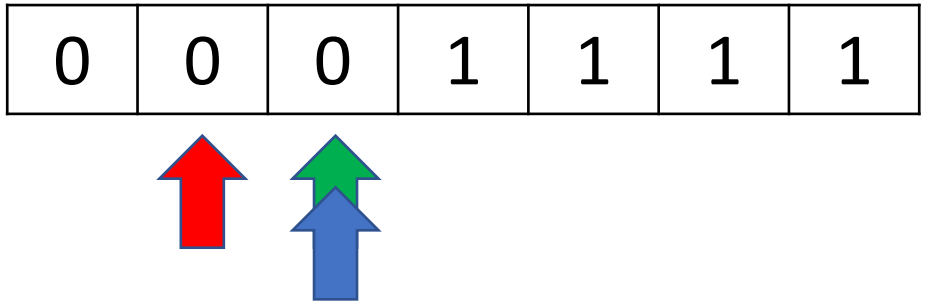

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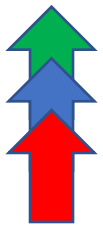

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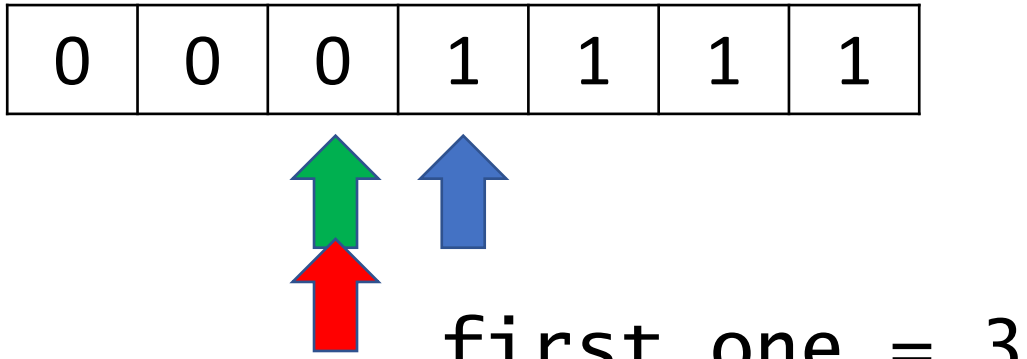


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7 – 3 = 4

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Which algorithm is best?

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Which algorithm is best?

How can we compare algorithms?

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Running time, accuracy, simplicity, memory requirements, communication requirements,…

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Running Time – The rough worst-case number of operations needed to run an algorithm.

Allows us to ignore hardware performance.

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items in a

$$1 + (1 + 1)n + 1$$

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items in a

$$1 + (1 + 1)n + 1 = 2n + 2 \in O(n)$$

Big-O Notation

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Running Time:

$O(\log n)$ – Binary Search

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CSCI 532

Goals:

- Learn algorithmic tools/techniques.
- Learn to evaluate algorithms.

What it is:

- Algorithmic thinking
- Mathematical proofs
- Application of data structures

What it is not:

- Programming
- An Introductory Course

Skills Check

- Not graded.
- Allows me to see where we are starting.