Approximation Algorithms
CSCI 432

Approximation Algorithms

Minimization problem:

$$\text{ALG} \leq \alpha \, \text{OPT}$$

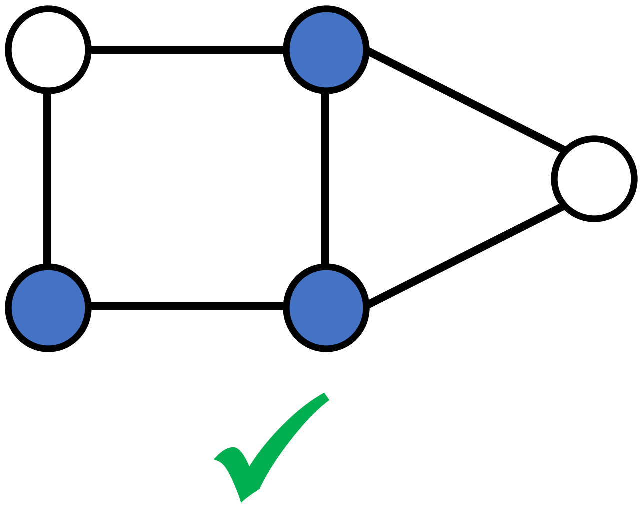
- ALG: Cost (size) of algorithm's solution.
- α: Approximation Ratio
- OPT: Cost (size) of optimal solution.

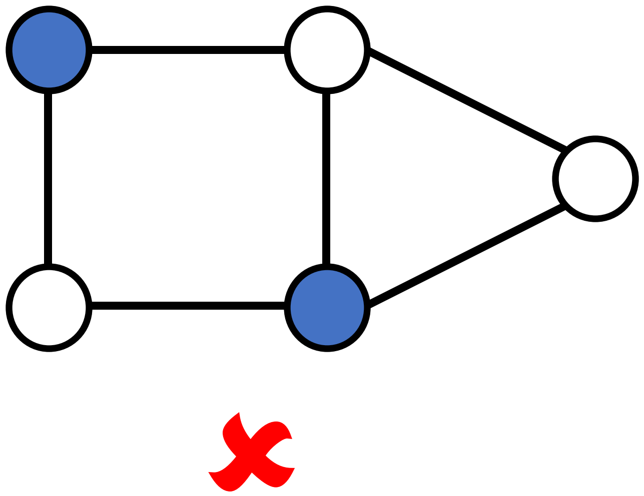
Maximization problem:

$$\text{ALG} \geq \frac{1}{\alpha} \text{OPT}$$

Vertex Cover

Vertex Cover: Given graph, find the smallest subset of vertices such that every edge in the graph has at least one vertex in the subset.





Vertex Cover

```
while uncovered edge exists
   select both vertices from uncovered edge
```

Consider a set of edges, $E' \subset E$, that do not share vertices. Is there a relationship between the minimum vertex cover and $|E'|$?

$$|E'| \leq \text{OPT}$$

Does the size of the algorithm's output relate to a set of edges that do not share vertices?

$$\text{ALG} = 2\,|E'|$$

$$\Longrightarrow \text{ALG} = 2\,|E'| \leq 2\ \text{OPT} \Longrightarrow \text{ALG} \leq 2\ \text{OPT}$$

Vertex Cover ILP

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$x_i \in \{0,1\}$ = Indicates if vertex i is selected.
Objective: $\min \sum_i x_i$
Subject to: $x_i + x_j \geq 1$, for each edge $e = (i, j)$

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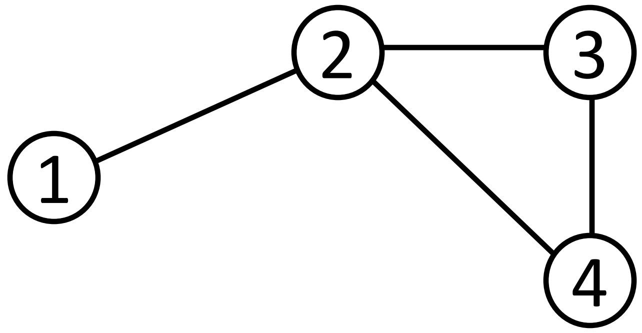
<u>Example:</u>

Objective: $\min x_1 + x_2 + x_3 + x_4$
Subject to:
$$x_1 + x_2 \geq 1$$
$$x_2 + x_3 \geq 1$$
$$x_2 + x_4 \geq 1$$
$$x_3 + x_4 \geq 1$$
$$x_1, x_2, x_3, x_4 \in \{0,1\}$$



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∈ NP-Complete

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LP Relaxation: Remove all integrality constraints to turn ILP into LP.

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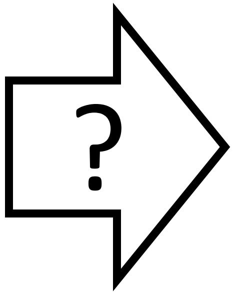

Vertex Selection

Vertex Cover ILP

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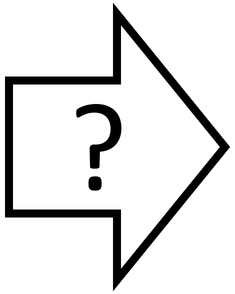

Vertex Selection

If $x_i = 1$, what should we do with vertex i?

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? → Vertex Selection

If $x_i = 1$, what should we do with vertex i? Add to subset S
If $x_i = 0$, what should we do with vertex i?

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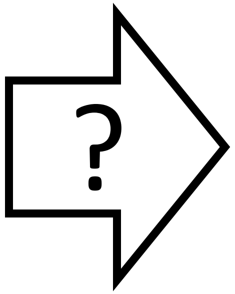

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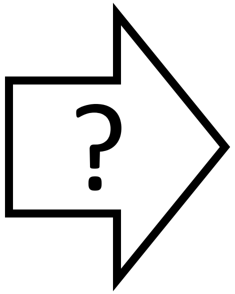

Vertex Selection

If $x_i = 1$, what should we do with vertex i? Add to subset S
If $x_i = 0$, what should we do with vertex i? Don't add to subset S
If $x_i = \frac{126}{337}$, what should we do with vertex i?

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If $x_i \geq \frac{1}{2}$, add vertex i to our subset S.

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Yes. For every edge, $x_i + x_j \geq 1$. Thus, at least one of x_i or $x_j \geq \frac{1}{2}$.

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Is S a vertex cover?

Yes. For every edge, $x_i + x_j \geq 1$. Thus, at least one of x_i or $x_j \geq \frac{1}{2}$. So for every edge, at least one of its vertices will be in S.

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What is the relationship between $\text{ALG} = |S|$ and OPT?

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Can we bound OPT from below?

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Let x_{ILP} and x_{LP} be the set of x values found by the ILP and LP

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Let x_{ILP} and x_{LP} be the set of x values found by the ILP and LP

Claim: $\sum x_{\text{LP}} \leq$ OPT.

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Proof: OPT $=$?

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Proof: OPT $= \sum x_{\text{ILP}}$, where $x_i \in \{0,1\}$...?

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Can we bound OPT from below?

Let x_{ILP} and x_{LP} be the set of x values found by the ILP and LP

Claim: $\sum x_{\text{LP}} \leq$ OPT.

Proof: OPT $= \sum x_{\text{ILP}}$, where $x_i \in \{0,1\}$. When x_i is relaxed so that $x_i \in [0,1]$, this gives more possibilities to further decrease $\sum_i x_i$. Thus, $\sum x_{\text{LP}} \leq$ OPT.

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Can we bound OPT from below?

Law of LP Relaxations:

$\text{OPT}_{\text{LP}} \leq \text{OPT}_{\text{ILP}}$ * **(minimization problem)**

***Objective values, not individual variable values.**

e ILP and LP

s

rt

decrease $\sum_i x_i$. Thus, $\sum x_{\text{LP}} \leq \text{OPT}$.

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How does $\sum x_{\text{LP}}$ relate to ALG?

$$\sum x_{\text{LP}} = \sum_{x_i \in x_{\text{LP}}} x_i \geq \sum_{x_i \in x_{\text{LP}}:\, x_i \geq \frac{1}{2}} x_i$$, because...?

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$$\geq \sum_{x_i \in x_{\text{LP}}:\, x_i \geq \frac{1}{2}} \frac{1}{2}, \text{ because...?}$$

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$$= \frac{1}{2} \left| \left\{ x_i \in x_{\text{LP}}:\ x_i \geq \frac{1}{2} \right\} \right|$$

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$$\sum x_{\text{LP}} \geq \frac{1}{2}\text{ALG and } \sum x_{\text{LP}} \leq \text{OPT}$$

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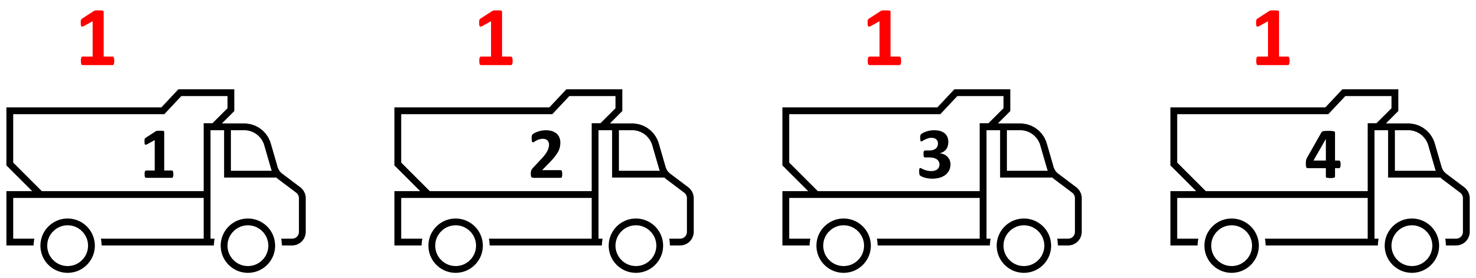
$\sum x_{\text{LP}} \geq \frac{1}{2}$ ALG and $\sum x_{\text{LP}} \leq$ OPT

ALG ≤ 2 OPT

Truck Loading Problem

Problem: Deliver n objects using the smallest number of trucks. Each object weighs between 0 and 1 ton. Each truck has a capacity of 1 ton.



Object	Weight
1	0.75
2	0.4
3	0.3
4	0.4
5	0.1

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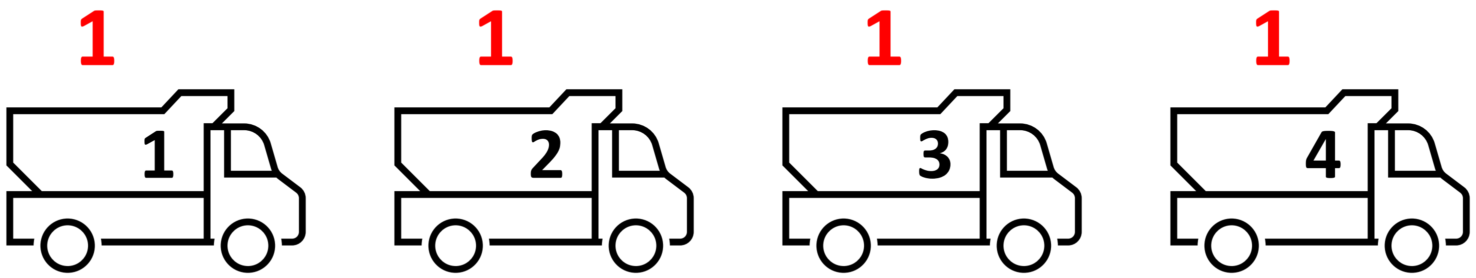
Algorithm: Line up trucks. For each object, place it on the first truck it fits on.



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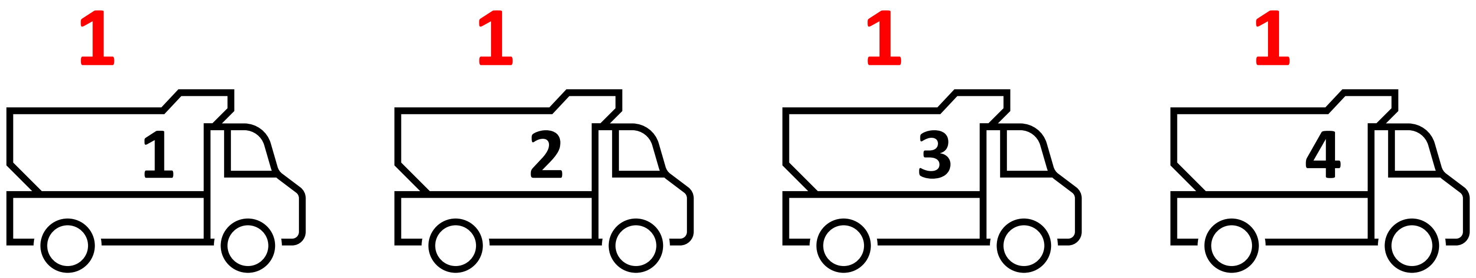

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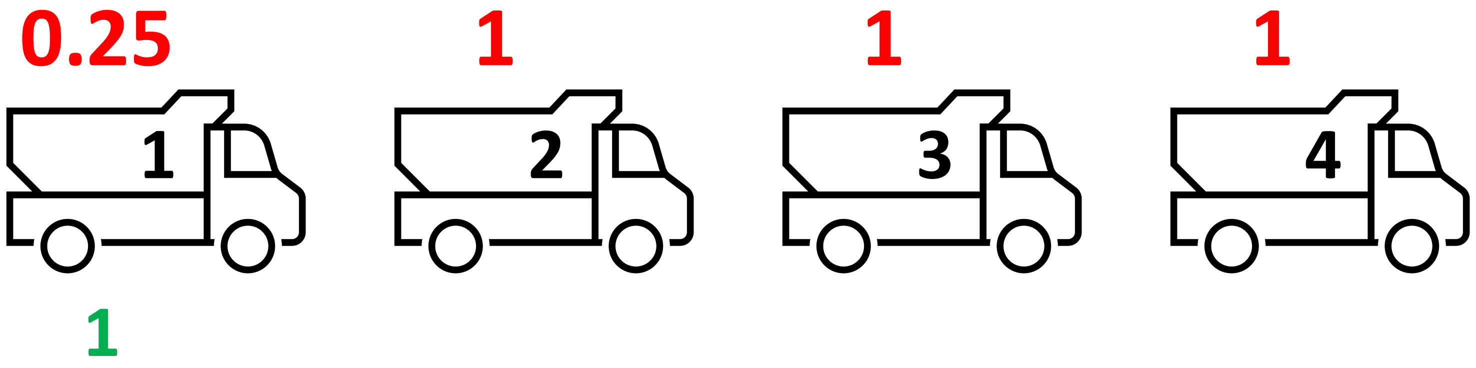

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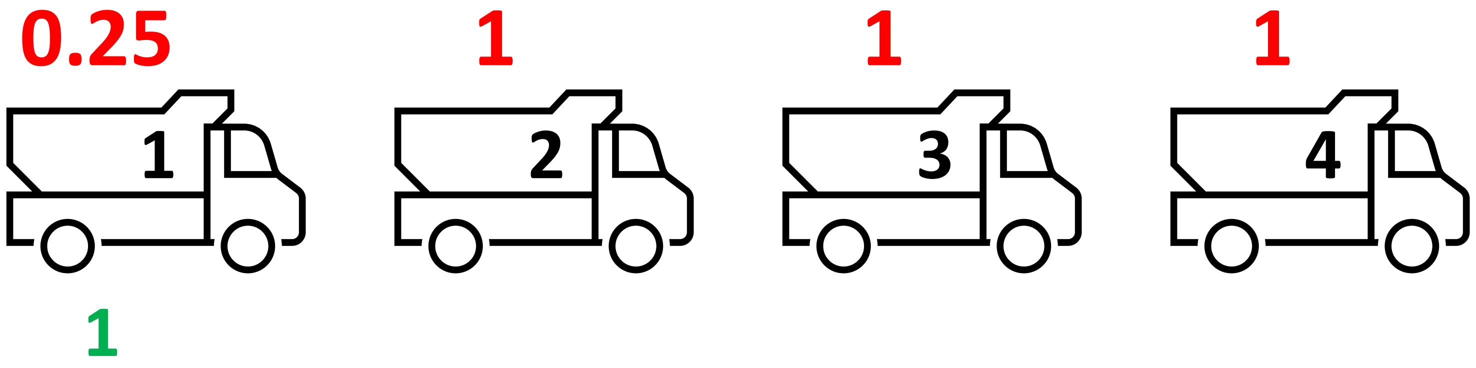

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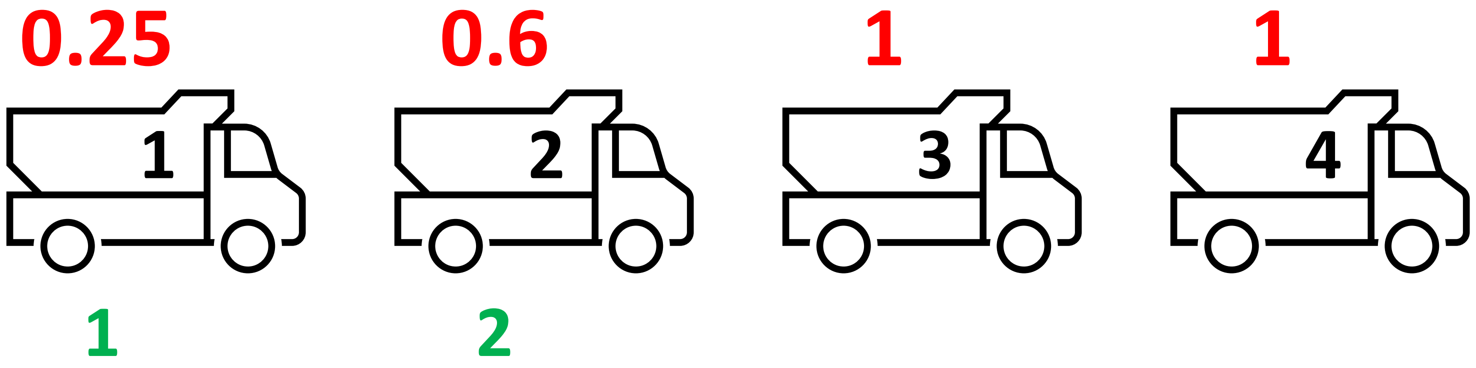

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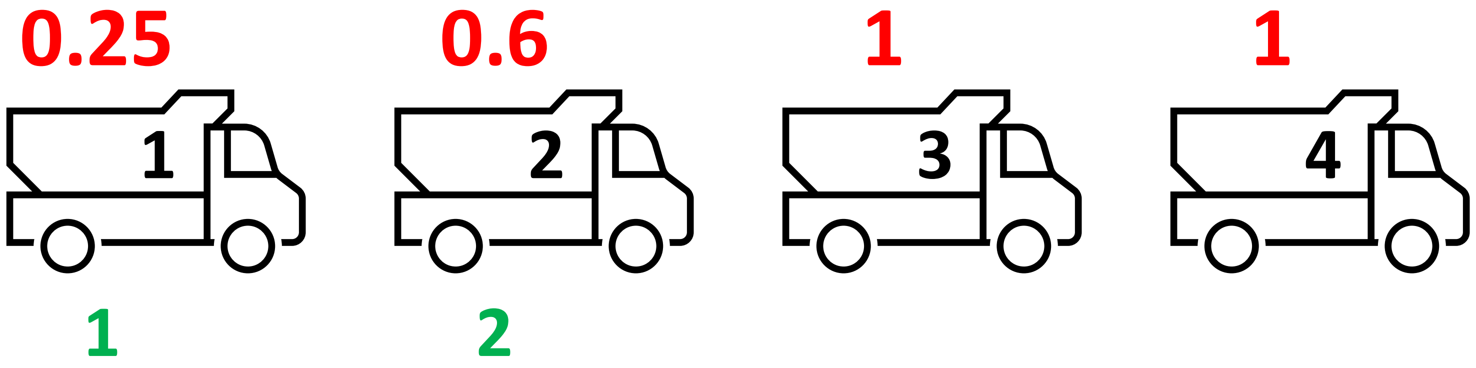

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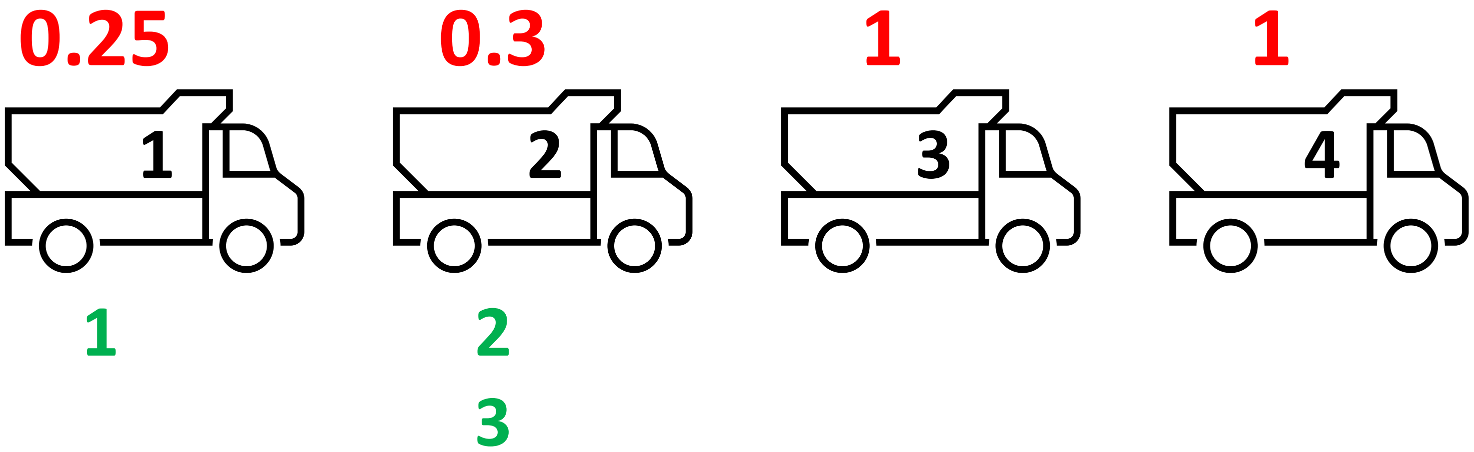

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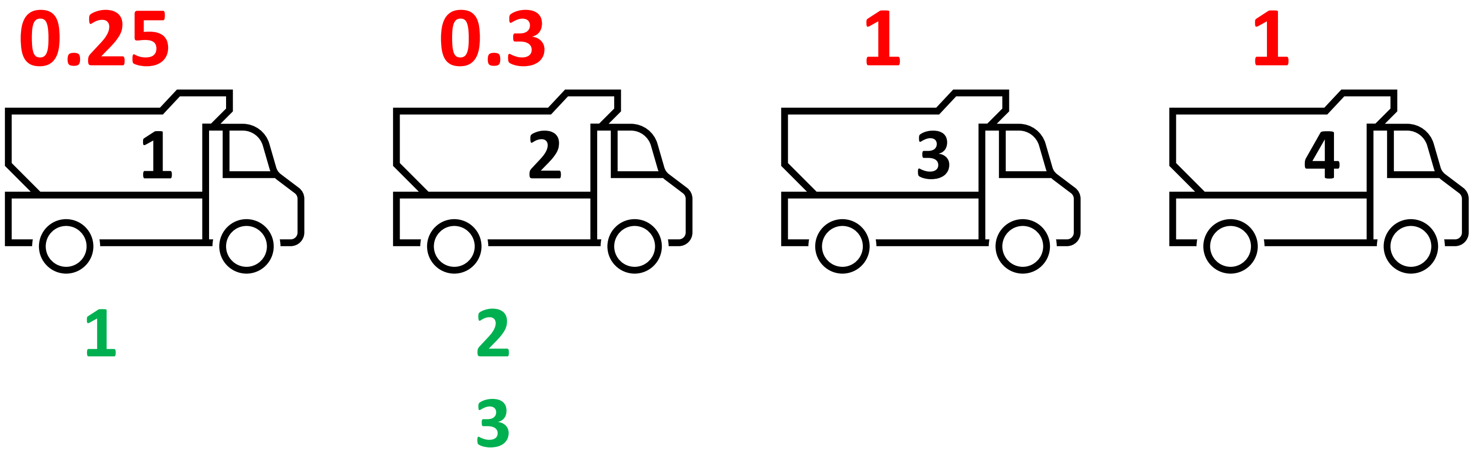

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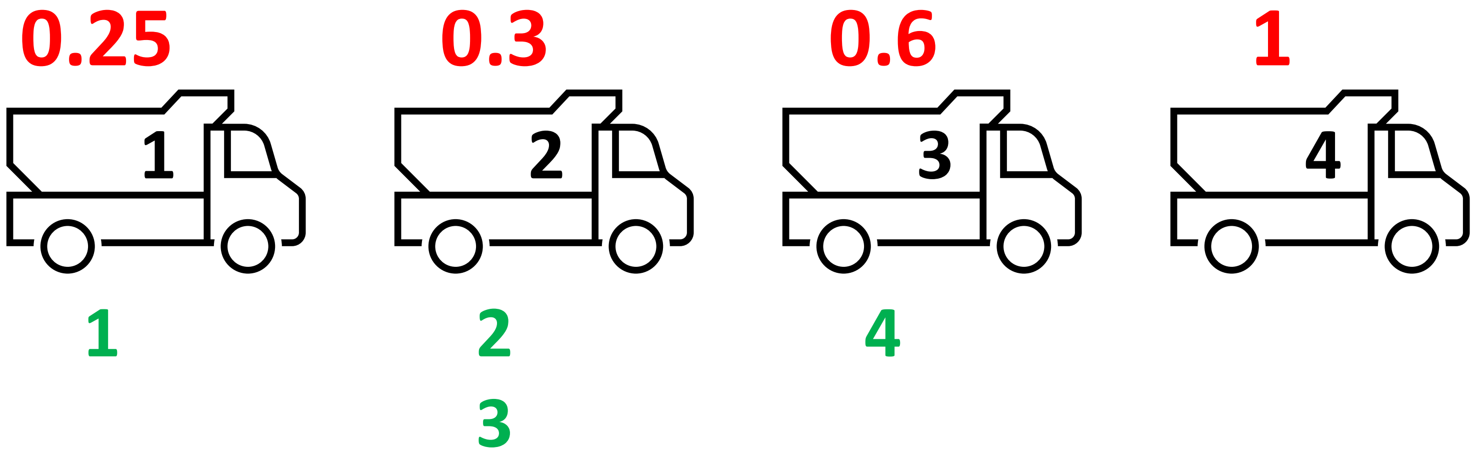

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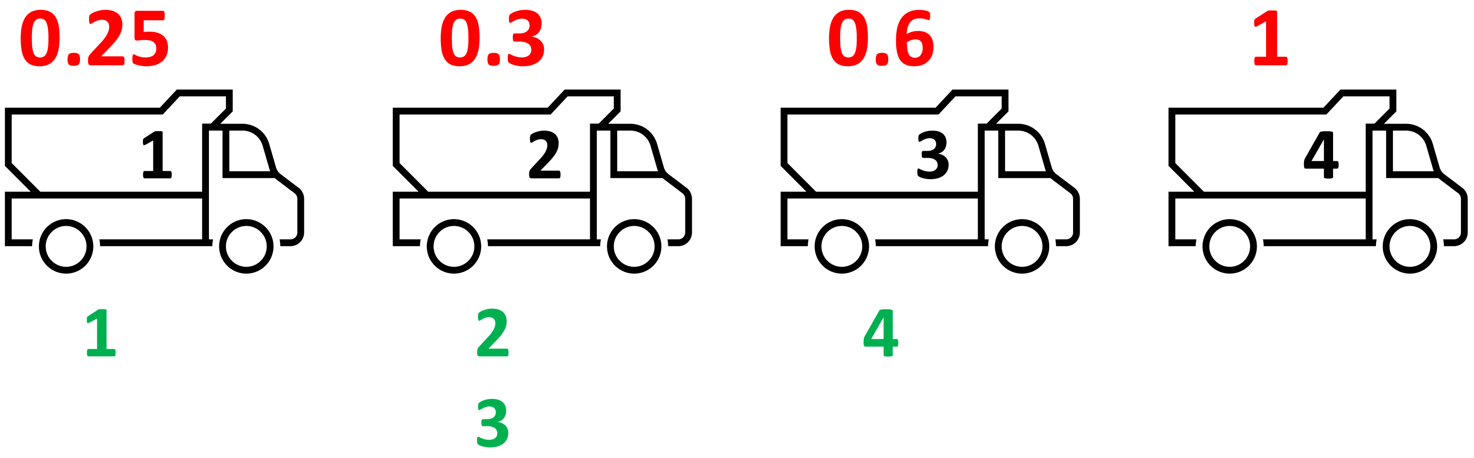

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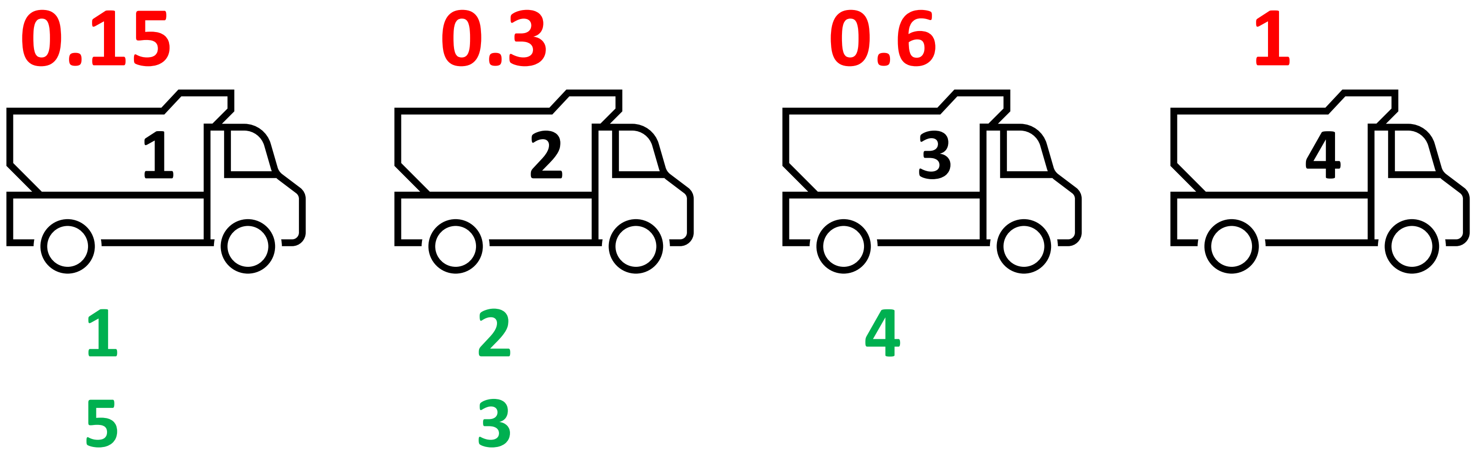

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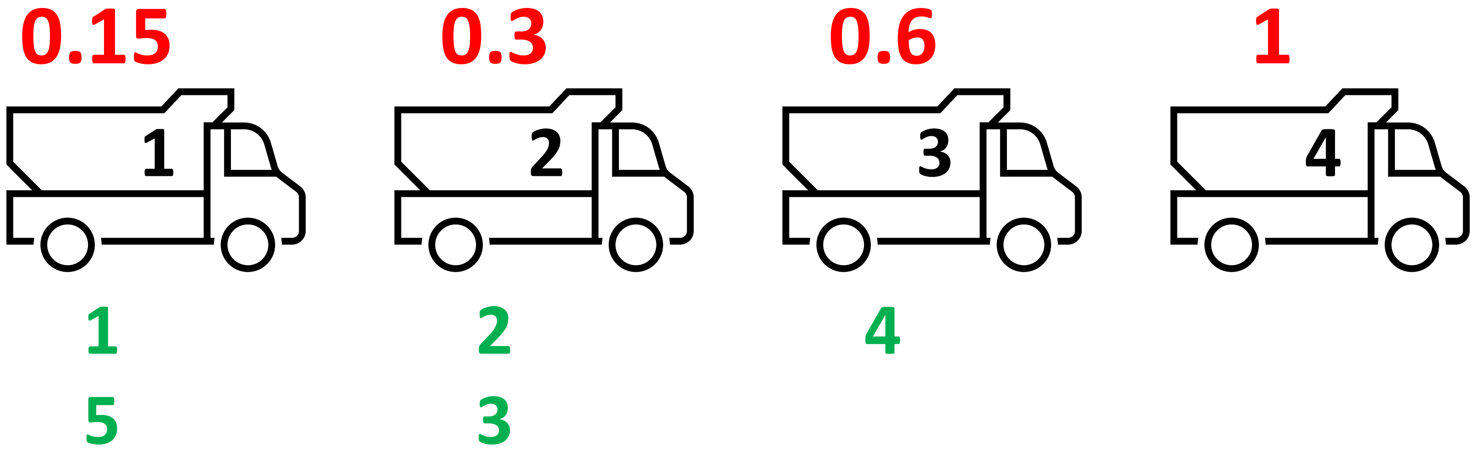

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Goal: Show this algorithm is 2-approximation algorithm.

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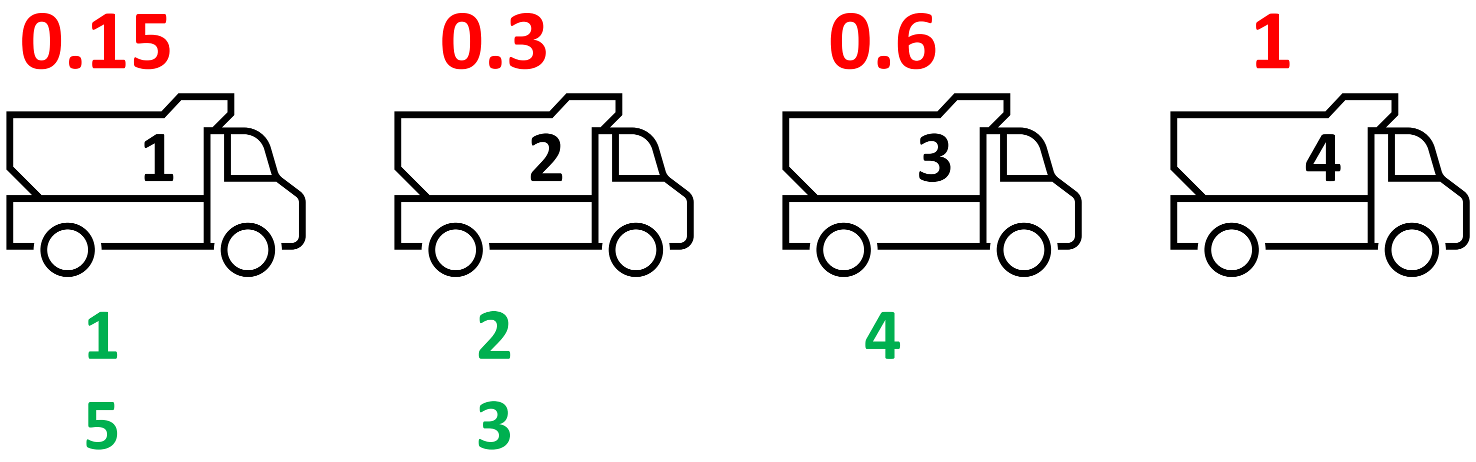

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Could we ever have a used truck that is less than half filled?

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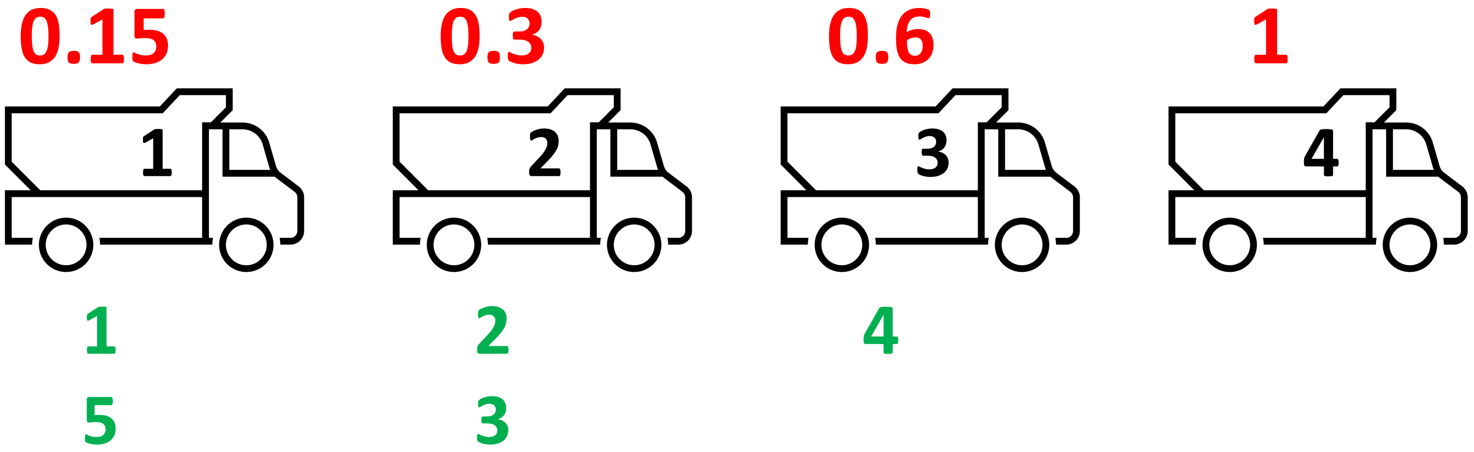

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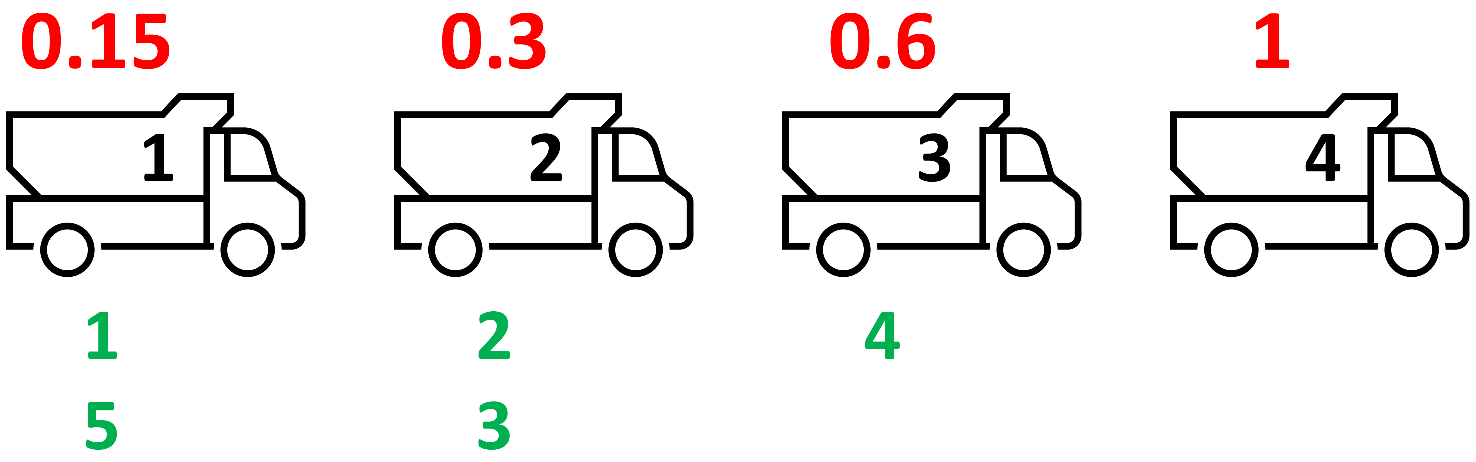

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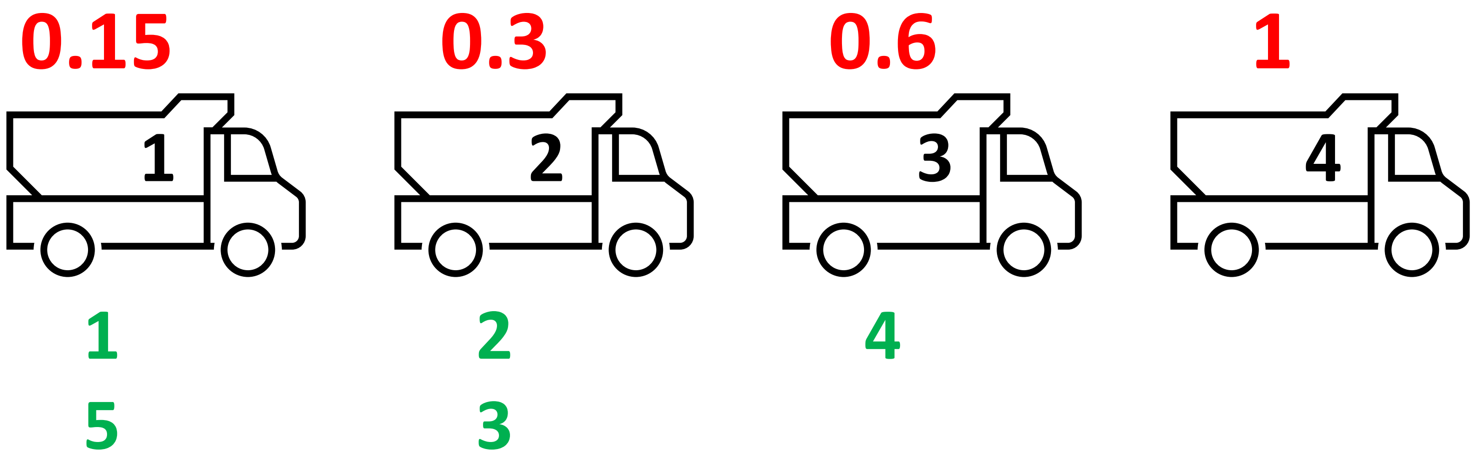

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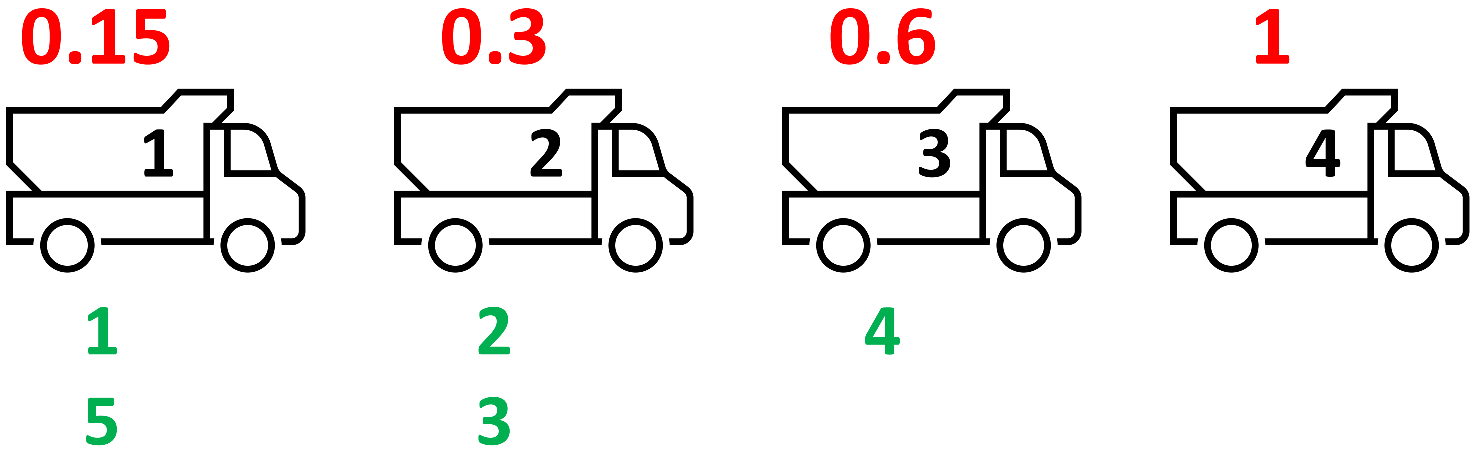

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How does W relate to ALG?

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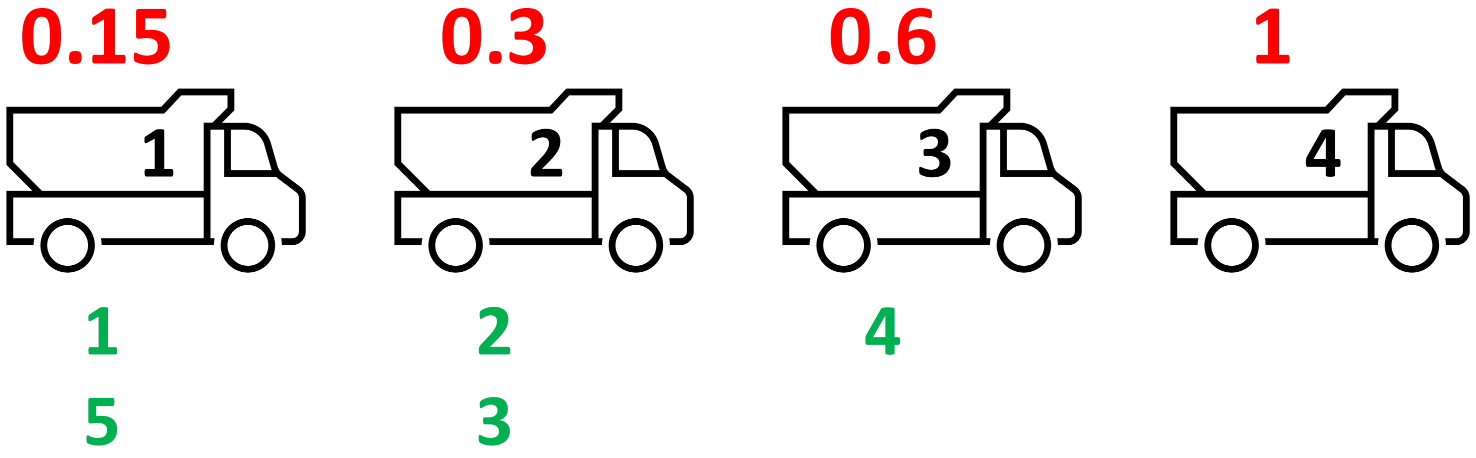

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$$\Rightarrow W > \frac{1}{2}(ALG - 1)$$

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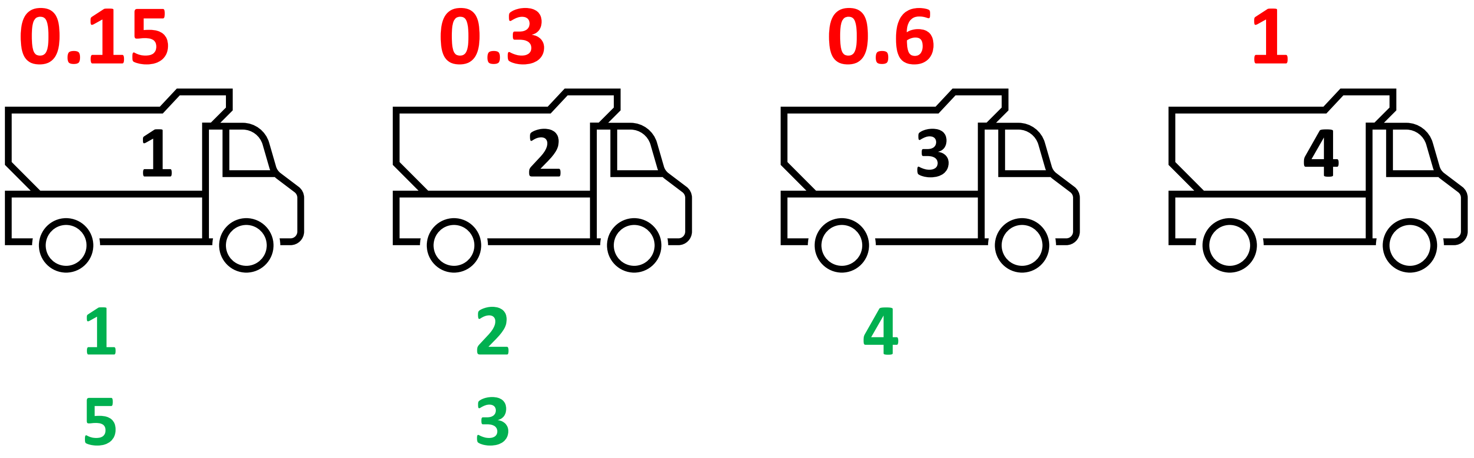

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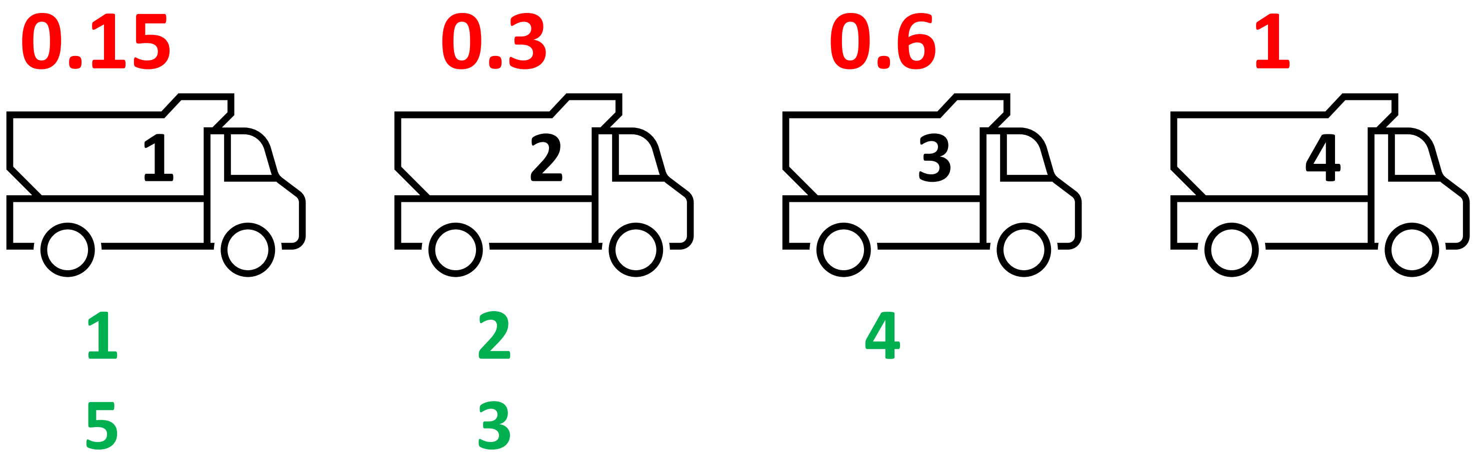

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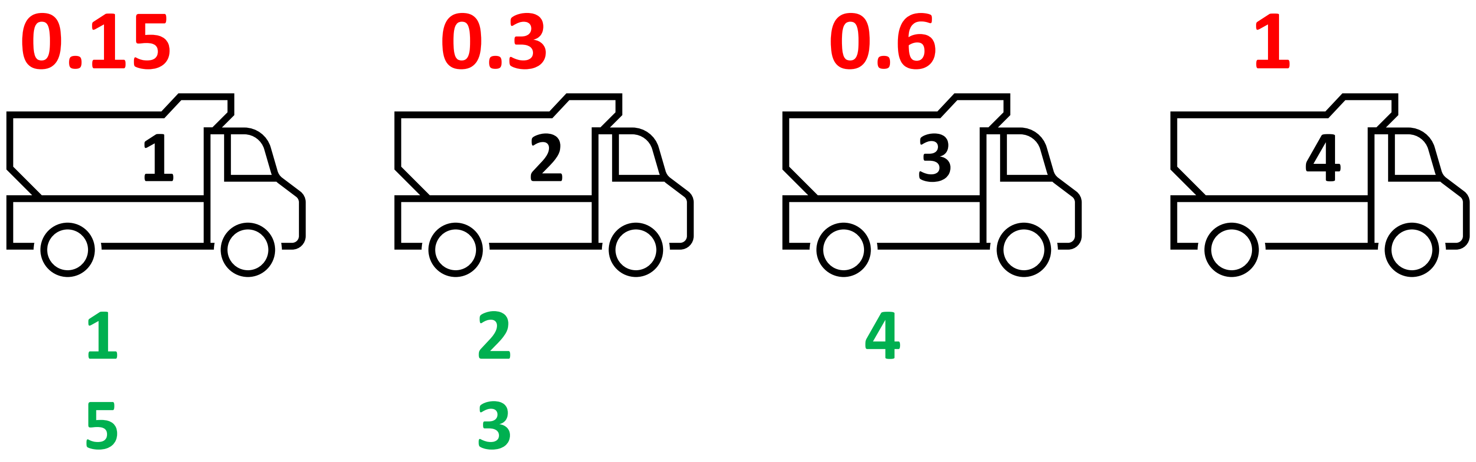

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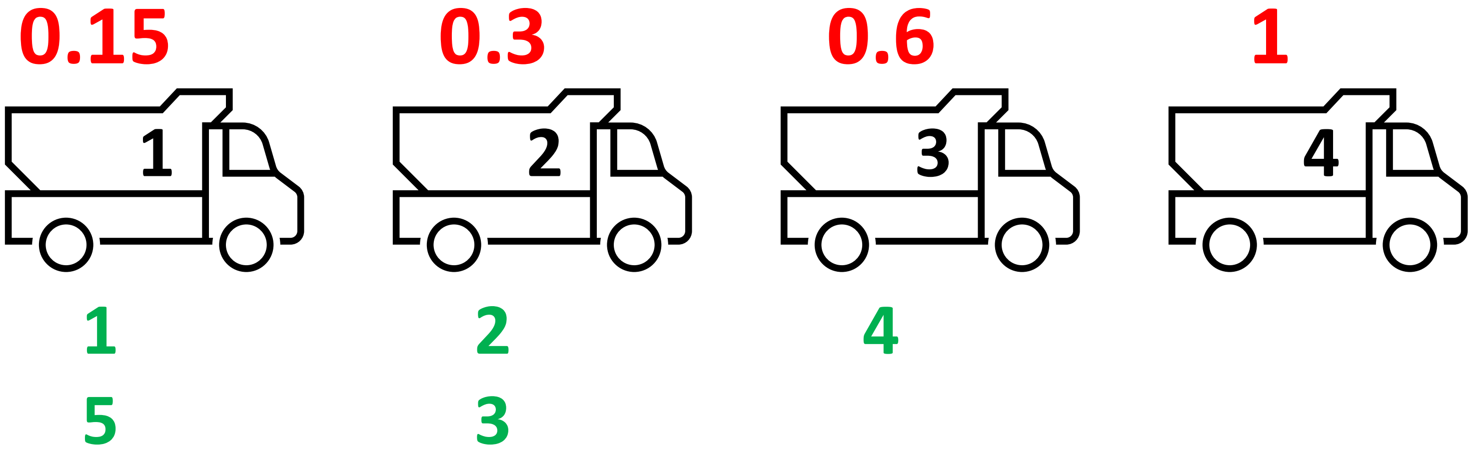

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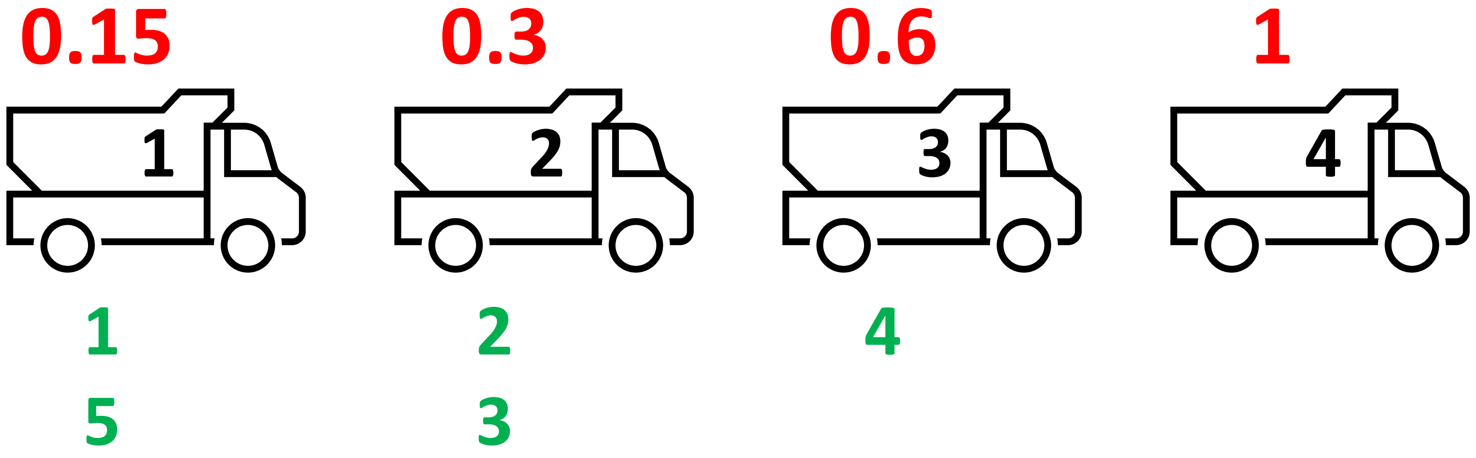

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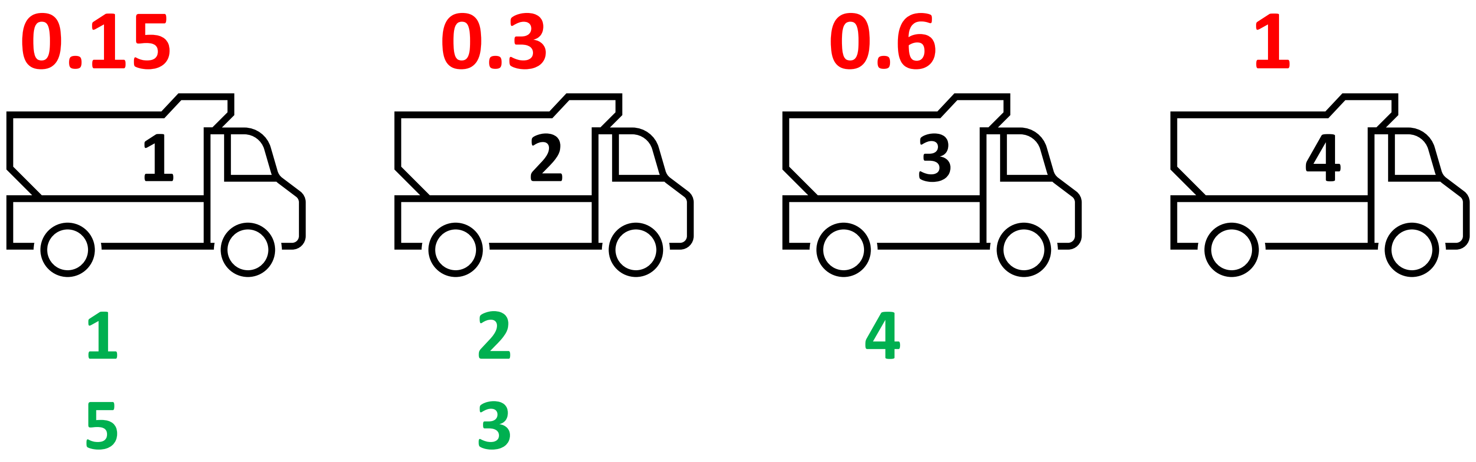

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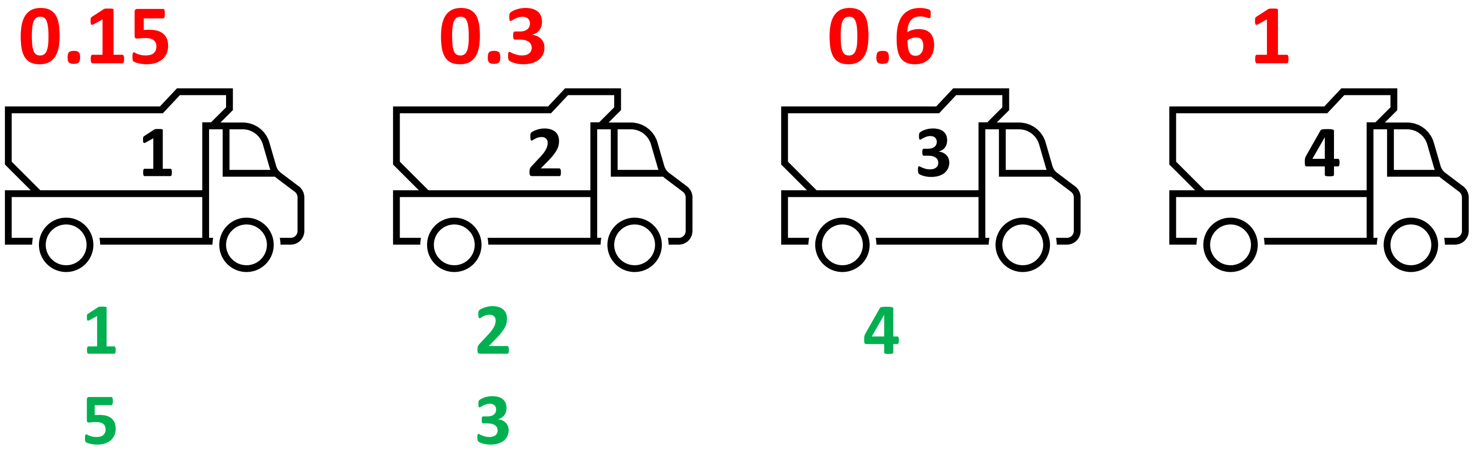

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$$ALG < 2\,OPT + 1 \Rightarrow ALG \leq 2\,OPT$$

ALG is an integer less than the integer $2\,OPT + 1$, so the most it could be is the integer $2\,OPT$.